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# Reaching nirvana with a defaultable asset?

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## Notes

1. Cox and Huang ([1989](#)) require the global Lipschitz continuity of the diffusive coefficient for the risky asset-value process [see Conditions A and B at p. 46 in Cox and Huang ([1989](#))]. By contrast, the diffusive coefficient in our setting is the square root of the risky asset value.
2. See, for instance, Davydov and Linetsky ([2001](#)), p. 952, first paragraph, with  $(S_t = P_t)$  and  $(p = \frac{1}{2})$ .

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7. Similar investment-horizon effects have been found in the literature on dynamic portfolio choice with a risky non-defaultable asset characterized by a mean-reverting drift and a constant volatility [see, for example, Koijen et al. (2009)].
8. The functions  $U$  and  $V$  are conjugate if and only if  $(U(w)=\inf_{y>0}(V(y)+wy))$  and  $(V(y)=\sup_{w>0}(U(w)-wy))$ .
9. The superscript  $(\left( \cdot \right)^{\{DS\}})$  refers to the notations of Delbaen and Shirakawa (2002), whereas  $a$ ,  $b$ ,  $c$  are defined in the statement of Proposition 3.2.

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$$u(w)=\sup_{\{\tilde{W}\}\in \{\mathcal{W}\}} E[U(\tilde{W}_T)].$$

(A.1)

We apply to problem (A.1) the duality approach developed by Kramkov and Schachermayer (1999, 2003). To this aim, we observe that the utility function  $U$  satisfies Inada conditions [equation (2.4) in Kramkov and Schachermayer (1999)]. Let  $V$  denote the conjugate function<sup>8</sup> of  $U$ , that is  $(V(y)=\frac{\phi}{1-\phi}y^{-\frac{1-\phi}{\phi}})$  and define

$$v(y)=E\left[ V\left( y\eta \right) \right]$$

where  $(\eta )$  is given by (2.3). Kramkov and Schachermayer (2003) show that if  $(v(y)<\infty )$  for all  $(y>0)$ , then  $(u(w)<\infty )$  for all  $(w>0)$  and  $u$  and  $v$  are

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$$\begin{aligned}
 U(\{\tilde{W}\}^*)\right] &= U(w e^{\{rT\}})E\left[\left(\frac{\eta^{-\frac{1}{\phi}}}{\eta^{-\frac{1-\phi}{\phi}}}\right)^{1-\phi}\right] \\
 &= U(w e^{\{rT\}})\left(E\left[\eta^{-\frac{1-\phi}{\phi}}\right]\right)^{\phi} \\
 &= U(w e^{\{rT\}})\exp\left(\frac{1-\phi}{\phi}\int_0^T Y_t dZ_t + \frac{1-\phi}{2\phi}\int_0^T Y_t dt\right) \\
 &= U(w e^{\{rT\}})F(T,y).
 \end{aligned}$$

$\square$

## 1.2 A.2 Proof of proposition [3.2](#)

Since  $(a=\frac{1-\phi}{\phi},)$  then

$$\begin{aligned}
 &E\left[\exp\left(\frac{1-\phi}{\phi}\int_0^T \sqrt{Y_t} dZ_t + \frac{1-\phi}{2\phi}\int_0^T Y_t dt\right)\right] \\
 &= E\left[\exp\left(\frac{1-\phi}{\phi}\int_0^T \sqrt{Y_t} dZ_t + \frac{1-\phi}{2\phi}\int_0^T Y_t dt\right)\right]
 \end{aligned}$$

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Nikodym density of an equivalent probability measure  $\mathbb{Q}$  equivalent to  $\mathbb{P}$ .) Girsanov's theorem implies then that

$$\begin{aligned} \hat{Z}_t &= Z_t - \int_0^t a \sqrt{Y_s} ds \\ \end{aligned}$$

(A.3)

is a  $\mathbb{Q}$ -Brownian motion. Thus,

$$\begin{aligned} F(T,y) &= \left( E^{\mathbb{Q}} \left[ \exp \left( \frac{a^2 + a}{2} \int_0^T Y_t dt \right) \right] \right)^{\phi}, \\ \end{aligned}$$

(A.4)

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$$M_{\{T\}} = \exp \left( \frac{a^{\{2\}} + a^{\{2\}}}{2} \int_0^T Y_{\{t\}} dt \right) \quad G(0, Y_{\{T\}}) = \exp \left( \frac{a^{\{2\}} + a^{\{2\}}}{2} \int_0^T Y_{\{t\}} dt \right), \quad \text{\end{aligned}}$$

since  $(G(0, \cdot) = 1)$ . The martingality condition  $(M_{\{0\}} = E^{\{\hat{\mathbb{Q}}\}}[M_{\{T\}}])$  yields then

$$G(T, y) = E^{\{\hat{\mathbb{Q}}\}} \left[ \exp \left( \frac{a^{\{2\}} + a^{\{2\}}}{2} \int_0^T Y_{\{s\}} dt \right) \right], \quad \text{\end{aligned}}$$

that allows us to find (A.4). By imposing 0 drift on the Ito decomposition under  $(\hat{\mathbb{Q}})$  of the process  $(M_{\{t\}})$  we get the partial differential equation for  $G$

$$\left\{ \begin{array}{l} G_{\{t\}} = \frac{\xi^{\{2\}}}{2} \end{array} \right.$$

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$(M_t)$  is a martingale we define

$$Z_t = \frac{M_t}{M_0},$$

which is a  $(\hat{\mathbb{Q}})$  local martingale as well, and show that  $(Z_t)$  is a  $(\hat{\mathbb{Q}})$  martingale. To this aim, we first observe that process  $(Z_t)$  is a stochastic exponential. In fact, Ito formula implies that

$$dM_t = e^{\frac{a^2}{2} + a \int_0^t Y_s ds} \frac{\partial}{\partial y} G(T-t, Y_t) \sqrt{c} \phi(Y_t) d\hat{Z}_t$$

from the dynamics of  $Y$  with respect to  $(\hat{\mathbb{Q}})$  in Eq. (A.5). Since  $\frac{\partial}{\partial y} G(T-t, Y_t) = e^{Y_t} g(T-t) \cdot g(T-t) = G(T-$

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We apply Theorem 4.1 in Klebaner and Lipster ([2014](#)) to conclude that  $\{\frac{z_t}{t}\}$  is a true martingale. In particular, with Klebaner and Lipster notations (4.2) at page 44

$$\begin{aligned} a_s(y) &= by \quad b_s(y) = 2\sqrt{c\phi} \sqrt{y} \quad \text{from (A.5)} \\ \sigma_s(y) &= 2\sqrt{c\phi} g(T-s)\sqrt{y} \quad \text{from our def. of } m_t, \end{aligned}$$

we get

$$\begin{aligned} L_s(y) &= 2ya_s(y) + (b_s(y))^2 \\ &= 2by^2 + 4c\phi y \quad \mathcal{L}_s(y) = 2y[a_s(y) + b_s(y)\sigma_s(y) + (b_s(y))^2] \\ &= 2by^2 + 8c\phi y^2 g(T-s) + 8c\phi y^2 \end{aligned}$$

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denote with  $(Z_t^{\mathbb{Q}})$  the  $\mathbb{Q}$ -Brownian motion

$$\begin{aligned} Z_t^{\mathbb{Q}} &= Z_t + \int_0^t \sqrt{Y_s} ds. \end{aligned}$$

(A.10)

Before proving the result, we first list some technical lemmas.

### Lemma A.1

Let  $(L^* = \frac{L_T}{\eta})$  where  $(L_T)$  is given by (A.2). Then  $(L_t^* = E^{\mathbb{Q}}[\left. L^* \right| \mathcal{F}_t])$  satisfies the stochastic differential equation

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and from the definitions of  $\eta$  in (2.3) and of  $L$  in (A.2) that

$$\begin{aligned} L_T^* &= \frac{L_T}{\eta} = \exp \left( a \int_0^T \sqrt{Y_s} \, dZ_s - \frac{a^2}{2} \int_0^T Y_s \, ds + \int_0^T \sqrt{Y_s} \, dZ_s + \frac{1}{2} \int_0^T Y_s \, ds \right) \\ &= \exp \left( \left( a+1 \right) \int_0^T \sqrt{Y_s} \, dZ_s + \frac{1-a^2}{2} \int_0^T Y_s \, ds \right). \end{aligned}$$

From the definition of  $(Z_s^{\mathbb{Q}})$  in Eq. (A.10) we get

$$\begin{aligned} L_T^* &= \exp \left( \left( a+1 \right) \int_0^T \sqrt{Y_s} \, dZ_s^{\mathbb{Q}} - \frac{1-a^2}{2} \int_0^T Y_s \, ds \right) \\ &= \exp \left( \left( a+1 \right) \int_0^T \sqrt{Y_s} \, dZ_s^{\mathbb{Q}} - \frac{\left( a+1 \right)^2}{2} \int_0^T Y_s \, ds \right). \end{aligned}$$

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$$M_{\{t\}}=e^{\{\frac{a^{\{2\}}+a\}{2}\}\int_{\{0\}}^{\{t\}}Y_{\{s\}}ds}e^{\{Y_{\{t\}}g(T-t)\}}.$$

Then we have

$$dM_{\{t\}}\&=2\sqrt{c\phi Y_{\{t\}}}M_{\{t\}}g(T-t)d\hat{Z}_{\{t\}}\&=2\sqrt{c\phi Y_{\{t\}}}M_{\{t\}}g(T-t)\left(dZ_{\{t\}}^{\{\mathbb{Q}\}}-\frac{1}{\phi}\sqrt{Y_{\{t\}}}dt\right)$$

(A.12)

where  $(\hat{Z}_{\{t\}})$  is defined in (A.3) and  $(Z_{\{t\}}^{\mathbb{Q}})$  in Eq. (A.10).

## Proof of the Lemma

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since  $(d\tilde{P})_t = \tilde{P}_t \sqrt{Y_t} \sigma_t dt + \tilde{P}_t \sigma_t dZ_t = \tilde{P}_t \frac{\xi}{\sqrt{Y_t}} d\mathbb{Q}$  from (2.1) and (A.10).

Therefore, we look for the Ito representation of  $(\tilde{W})^{*}(t) = E^{\mathbb{Q}}[(\tilde{W})^{*} | \mathcal{F}_t]$  to derive  $(\psi^{*})$ . Denoting with  $(L_t = E[\tilde{L}_T | \mathcal{F}_t])$  and  $(\eta_t = E[\eta | \mathcal{F}_t])$ , we have

$$\begin{aligned} E^{\mathbb{Q}}[\eta^{1-\frac{1}{\phi}} | \mathcal{F}_t] &= \frac{E[\eta^{1-\frac{1}{\phi}} | \mathcal{F}_t]}{E[\eta^{1-\frac{1}{\phi}}]} \quad (\text{by Bayes' rule}) \\ &= \frac{E[\exp(\int_0^T \sqrt{Y_t} dZ_t + \frac{a}{2} \int_0^T Y_t dt) | \mathcal{F}_t]}{E[\exp(\int_0^T \sqrt{Y_t} dZ_t + \frac{a}{2} \int_0^T Y_t dt)]} \end{aligned}$$

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because  $(G(T,y)=\left( F(T,y)\right) ^{\frac{1}{\phi }}=\{E\left[ \eta ^{-\frac{1-\phi }{\phi }}\right] .\})$  It follows that the differential of the  $(\mathbb {Q})$ -martingale  $(\{\tilde{W}\}_t^*)$  is given by

$$\begin{aligned} d\{\tilde{W}\}_t^*&=\frac{w}{G(T,y)}d\left( L_t^*M_t\right) \&=\frac{w}{G(T,y)}L_t^*M_t\left[ \frac{1}{\phi }+2\sqrt{c\phi }g(T-t)\right] \sqrt{Y_t}dZ_t^{\mathbb {Q}}\quad \hbox{by}\left( \text{A}.11\right) \text{ and }\left( \text{A}.12\right) \&=\\ &\{\tilde{W}\}_t^*\left[ \frac{1}{\phi }+2\sqrt{c\phi }g(T-t)\right] \sqrt{Y_t}dZ_t^{\mathbb {Q}} \end{aligned}$$

Comparing this equation with Eq. (A.13), we obtain

$$\frac{\xi \psi _t^*}{\sqrt{Y_t}}=W^*(t)\left[ \frac{1}{\phi }+2\sqrt{c\phi }g(T-t)\right] \sqrt{Y_t} \end{aligned}$$

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