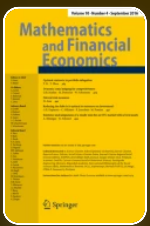
# Impact of contingent payments on systemic risk in financial networks

Published: 18 February 2019
Volume 13, pages 617–636, (2019) Cite this article



Mathematics and Financial Economics
Aims and scope →
Submit manuscript →

Tathagata Banerjee[1] & Zachary Feinstein orcid.org/0000-0002-6733-5724[1]



## Abstract

In this paper we study the implications of contingent payments on the clearing wealth in a network model of financial contagion. We consider an extension of the Eisenberg–Noe financial contagion model in which the nominal interbank obligations depend on the wealth of the firms in the network. We first consider the problem in a static framework and develop conditions for existence and uniqueness of solutions as long as no firm is speculating on the failure of other firms. In order to achieve existence and uniqueness under more general conditions, we introduce a dynamic framework. We demonstrate how this dynamic framework can be applied to problems that were ill-defined in the static framework.

# References

1. Acharya, V., Bisin, A.: Counterparty risk externality: centralized versus over-the-counter markets. J. Econ. Theory **149**, 153–182 (2014)

Article MathSciNet MATH Google Scholar

2. Aliprantis, C.D., Border, Kim C.: Infinite Dimensional Analysis: A Hitchhiker's Guide. Springer, New York (2007)

MATH Google Scholar

3. Amini, H., Filipović, D., Minca, A.: Systemic risk with central counterparty clearing. Swiss Finance Institute Research Paper No. 13-34, Swiss Finance Institute (2015)

4. Amini, H., Filipović, D., Minca, Andreea: Uniqueness of equilibrium in a payment system with liquidation costs. Oper. Res. Lett. **44**(1), 1–5 (2016)

Article MathSciNet MATH Google Scholar

5. Awiszus, K., Weber, S.: The joint impact of bankruptcy costs, cross-holdings and fire sales on systemic risk in financial networks. Probab. Uncertain. Quant. Risk **2**(9), 1–38 (2017)

MathSciNet Google Scholar

6. Banerjee, T., Bernstein, A., Feinstein, Z.: Dynamic Clearing and Contagion in Financial Networks. In: Working paper (2018)

7. Barucca, P., Bardoscia, M., Caccioli, F., D'Errico, M., Visentin, G., Battiston, S., Caldarelli, G.: Network Valuation in Financial Systems. In: Working paper (2016)

8. Blanchet, J., Shi, Y.: Stochastic Risk Networks: Modeling, Analysis and Efficient Monte Carlo. Working paper (2012)

9. Capponi, A., Chen, P.-C.: Systemic risk mitigation in financial networks. J. Econ. Dyn. Control **58**, 152–166 (2015)

Article MathSciNet MATH Google Scholar

10. Capponi, A., Chen, P.-C., Yao, D.D.: Liability concentration and systemic losses in financial networks. Oper. Res. **64**(5), 1121–1134 (2016)

Article MathSciNet MATH Google Scholar

11. Chen, N., Liu, X., Yao, D.D.: An optimization view of financial systemic risk modeling: the network effect and the market liquidity effect. Oper. Res. **64**(5), 1089–1108 (2016)

Article MathSciNet MATH Google Scholar

12. Cifuentes, R., Shin, H.S., Ferrucci, G.: Liquidity risk and contagion. J. Eur. Econ. Assoc. **3**(2–3), 556–566 (2005)

Article Google Scholar

13. Cont, R.: The end of the waterfall: default resources of central counterparties. J. Risk Manag. Financ. Inst. **8**(4), 365–389 (2015)

Google Scholar

14. Cont, R., Fournié, D.-A.: Functional Itô calculus and stochastic integral representation of martingales. Ann. Probab. **41**(1), 109–133 (2013)

Article MathSciNet MATH Google Scholar

15. Cont, R., Minca, A.: Credit default swaps and systemic risk. Ann. Oper. Res. **247**, 523–547 (2016)

Article MathSciNet MATH Google Scholar

16. Cont, R., Moussa, A., Santos, E.B.: Network structure and systemic risk in banking systems. In: Handbook on Systemic Risk, pp. 327–368. Cambridge University Press (2013)

17. Eisenberg, L., Noe, T.H.: Systemic risk in financial systems. Manag. Sci. **47**(2), 236–249 (2001)

Article MATH Google Scholar

18. Elliott, M., Golub, B., Jackson, M.O.: Financial networks and contagion. Am. Econ. Rev. **104**(10), 3115–3153 (2014)

Article Google Scholar

19. Elsinger, H.: Financial Networks, Cross Holdings, and Limited Liability, vol. 156. Österreichische Nationalbank (Austrian Central Bank), Wien-Alsergrund (2009)

Google Scholar

20. Elsinger, H., Lehar, A., Summer, M.: Risk assessment for banking systems. Manag. Sci. **52**(9), 1301–1314 (2006)

Article MATH Google Scholar

21. Feinstein, Z.: Financial contagion and asset liquidation strategies. Oper. Res. Lett. **45**(2), 109–114 (2017)

Article MathSciNet MATH Google Scholar

22. Feinstein, Z.: Obligations with physical delivery in a multi-layered financial network. In: Working Paper (2018)

23. Feinstein, Z., El-Masri, F.: The effects of leverage requirements and fire sales on financial contagion via asset liquidation strategies in financial networks. Stat. Risk Model **34**(3–4), 113–139 (2017)

MathSciNet MATH Google Scholar

24. Feinstein, Z., Pang, W., Rudloff, B., Schaanning, E., Sturm, S., Wildman, M.: Sensitivity of the Eisenberg–Noe clearing vector to individual interbank liabilities. SIAM J. Financ. Math. **9**(4), 1286–1325 (2018)

Article MathSciNet MATH Google Scholar

25. Feinstein, Z., Rudloff, B., Weber, S.: Measures of systemic risk. SIAM J. Financ. Math. **8**(1), 672–708 (2017)

Article MathSciNet MATH Google Scholar

26. Gai, P., Kapadia, S.: Contagion in financial networks. In: Bank of England Working Papers 383. Bank of England (2010)

27. Glasserman, P., Young, H.P.: How likely is contagion in financial networks? J. Bank. Finance **50**, 383–399 (2015)

Article Google Scholar

28. Heise, S., Kühn, R.: Derivatives and credit contagion in interconnected networks. Eur. Phys. J. B **85**(4), 115 (2012)

Article Google Scholar

29. Klages-Mundt, A., Minca, A.: Cascading losses in reinsurance networks. In: Working Paper (2018)

30. Kusnetsov, M., Veraart, L.A.M.: Interbank clearing in financial networks with multiple maturities. In: Working Paper (2018)

31. Leduc, Matt, Poledna, S., Thurner, S.: Systemic risk management in financial networks with credit default swaps. J. Netw. Theory Finance **3**(3), 19–39 (2017)

Google Scholar

32. Liu, M., Staum, J.: Sensitivity analysis of the Eisenberg–Noe model of contagion. Oper. Res. Lett. **35**(5), 489–491 (2010)

Article MathSciNet MATH Google Scholar

33. Markose, S.M., Giansante, S., Gatkowski, M., Shaghaghi, A.R.: Too interconnected to fail: Financial contagion and systemic risk in network model of CDS and other credit enhancement obligations of us banks. In: Technical Report DP 683, Economics Department, University of Essex (2010)

34. Milgrom, P., Roberts, J.: Comparing equilibria. Am. Econ. Rev. **84**(3), 441–459 (1994)

Google Scholar

35. Murphy, D.: The systemic risk of otc derivatives central clearing. J. Risk Manag. Financ. Inst. **5**(3), 319–334 (2012)

Google Scholar

36. Nier, E., Yang, J., Yorulmazer, T., Alentorn, A.: Network models and financial stability. J. Econ. Dyn. Control **31**(6), 2033–2060 (2007)

Article MATH Google Scholar

37. Paddrik, M.E., Rajan, S., Young, P.: Contagion in the CDS market. In: OFR Working Paper 16-12, Office of Financial Research (2016)

38. Puliga, M., Caldarelli, G., Battiston, S.: Credit default swaps networks and systemic risk. Sci. Rep. **4**, 6822 (2014)

Article Google Scholar

39. Rogers, L.C.G., Veraart, L.A.M.: Failure and rescue in an interbank network. Manag. Sci. **59**(4), 882–898 (2013)

Article Google Scholar

40. Schuldenzucker, S., Seuken, S., Battiston, S.: Default ambiguity: credit default swaps create new systemic risks in financial networks. In: Working Paper (2017)

41. Schuldenzucker, S., Seuken, S., Battiston, S.: Finding clearing payments in financial networks with credit default swaps is PPAD-complete. In: Papadimitriou, C.H. (ed), 8th Innovations in Theoretical Computer Science Conference (ITCS 2017), volume 67 of Leibniz International Proceedings in Informatics (LIPIcs), pp. 32:1–32:20, Dagstuhl, Germany, (2017). Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik

42. Upper, C.: Simulation methods to assess the danger of contagion in interbank markets. J. Financ. Stab. **7**(3), 111–125 (2011)

Article Google Scholar

43. Veraart, L.A.M.: Distress and default contagion in financial networks. In: Working paper (2018)

## Author information

### Authors and Affiliations

**Department of Electrical and Systems Engineering, Washington University in St. Louis, St. Louis, MO, 63130, USA**

Tathagata Banerjee & Zachary Feinstein

## Corresponding author

Correspondence to Zachary Feinstein.


## Additional information

### Publisher's Note



## A Proof of Proposition 3.17

### Proof

Firstly, as in (6), the clearing wealths as a function of initial endowments are defined by

$$\begin{aligned} V(x) = x + \Pi (V(x))^\top [\bar{p}(V(x)) - V(x)^-]^+ - \bar{p}(V(x)). \end{aligned}$$

We will prove continuity by utilizing the closed graph theorem (see, e.g., [2, Theorem 2.58]) noting that Proposition 3.6 provides us with the condition that the clearing wealths map into a compact set. Theorem 4 of [34] immediately provides the monotonicity of the clearing wealths.

Fix $x \in \mathbb {R}^{n+1}_+$ and let $\mathcal {X}= x + [-1,1]^{n+1}$ be a closed compact neighborhood of $x$ in the full Euclidean space $\mathbb {R}^{n+1}$. Then we can define $V^x: \mathcal {X}\rightarrow \mathbb {R}^{n+1}$ as the restriction (and possible expansion to negative terms) of the domain of $V$ to $\mathcal {X}$. The graph of $V^x$ is given by:

$$\begin{aligned} {\text {graph}}V^x= & {} \Bigg \{(\hat{x},\hat{V}) \in \mathcal {X}\times \prod _{i \in \mathcal {N}_0} \left[ x_i-1-\sum _{j \in \mathcal {N}_0} \bar{L}_{ij} , x_i+1+\sum _{j \in \mathcal {N}} \bar{L}_{ji}\right] \; | \;

\hat{V}\\= & {} \hat{x} + \Pi (\hat{V})^\top [\bar{p}(\hat{V}) - \hat{V}^-]^+ - \bar{p}(\hat{V})\Bigg \}. \end{aligned}$$

To see that \({\text {graph}}V^x\) is closed let \((\hat{x}^k,\hat{V}^k)_{k \in \mathbb {N}} \subseteq {\text {graph}}V^x \rightarrow (\hat{x},\hat{V})\), then immediately

$$\begin{aligned} \hat{V}= & {} \lim _{k \rightarrow \infty } \hat{V}^k = \lim _{k \rightarrow \infty } \left[ \hat{x}^k + \Pi (\hat{V}^k)^\top [\bar{p}(\hat{V}^k) - (\hat{V}^k)^-]^+ - \bar{p}(\hat{V}^k)\right] \\= & {} \hat{x} + \Pi (\hat{V})^\top [\bar{p}(\hat{V}) - \hat{V}^-]^+ - \bar{p}(\hat{V}) \end{aligned}$$

by continuity of the nominal liabilities matrix $L$. Therefore by the closed graph theorem we immediately recover that \(V^x\) is continuous for any \(x \in \mathbb {R}^{n+1}_+\), which implies that $V$ is continuous at any $x$ as well and thus \(V: \mathbb {R}^{n+1}_+ \rightarrow \mathbb {R}^{n+1}\) is a continuous mapping. \(\square \)

## Rights and permissions



## About this article

### Cite this article

Banerjee, T., Feinstein, Z. Impact of contingent payments on systemic risk in financial networks. *Math Finan Econ* **13**, 617–636 (2019). https://doi.org/10.1007/s11579-019-00239-9

Received
25 May 2018

Accepted
11 February 2019

Published
18 February 2019

Issue Date
15 September 2019

DOI
https://doi.org/10.1007/s11579-019-00239-9

## Keywords

Systemic risk · Financial contagion · Insurance · Credit default swaps

## Mathematics Subject Classification

C62 · G22