The Path Integral Approach to Financial Modeling and Options Pricing

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Abstract

In this paper we review some applications of the path integral methodology of quantum mechanics to financial modeling and options pricing. A path integral is defined as a limit of the sequence of finite-dimensional integrals, in a much the same way as the Riemannian integral is defined as a limit of the sequence of finite sums. The risk-neutral valuation formula for path-dependent options contingent upon multiple underlying assets admits an elegant representation in terms of path integrals (Feynman–Kac formula). The path integral representation of transition probability density (Green's function) explicitly satisfies the diffusion PDE. Gaussian path integrals admit a closed-form solution given by the Van Vleck formula. Analytical approximations are obtained by means of the semiclassical (moments) expansion. Difficult path integrals are computed by numerical procedures, such as Monte Carlo simulation or deterministic discretization schemes. Several examples of path-dependent options are treated to illustrate the

theory (weighted Asian options, floating barrier options, and barrier options with ladder-like barriers).

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