An Asymptotic Expansion Approach to Pricing Financial Contingent Claims

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Abstract

We propose a new methodology for the valuation problem of financial contingent claims when the underlying asset prices follow a general class of continuous Itô processes. Our method can be applicable to a wide range of valuation problems including contingent claims associated with stocks, foreign exchange rates, the term structure of interest rates, and even their combinations. We illustrate our method by discussing the Black-Scholes economy when the underlying asset prices follow the continuous diffusion processes, which are not necessarily time-homogeneous. The standard Black-Scholes model on stocks and the Cox-Ingersoll-Ross model on the spot interest rate are simple examples. Then we shall give a series of examples on the valuation formulae including plain vanilla options, average options, and other contingent claims. We shall also give some numerical

evidence of the accuracy of the approximations we have obtained for practical purposes. Our approach can be rigorously justified by an infinite dimensional mathematics, the Malliavin-Watanabe-Yoshida theory recently developed in stochastic analysis.

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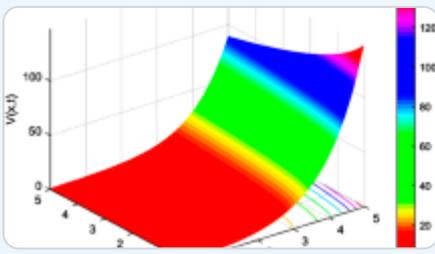
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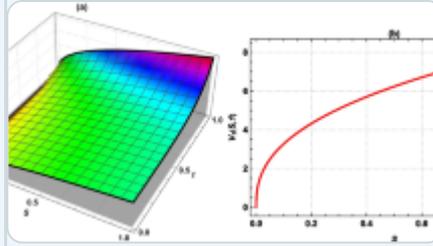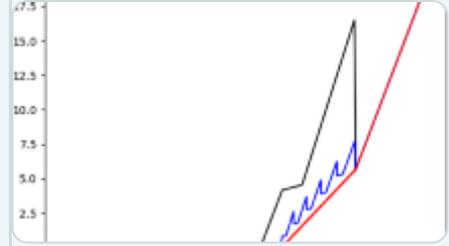
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