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An Asymptotic Expansion Approach to Pricing Financial Contingent Claims

| Published: June 1999

| Volume 6, pages 115–151, (1999) [Cite this article](#)



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Abstract

We propose a new methodology for the valuation problem of financial contingent claims when the underlying asset prices follow a general class of continuous Itô processes. Our method can be applicable to a wide range of valuation problems including contingent claims associated with stocks, foreign exchange rates, the term structure of interest rates, and even their combinations. We illustrate our method by discussing the Black-Scholes economy when the underlying asset prices follow the continuous diffusion processes, which are not necessarily time-homogeneous. The standard Black-Scholes model on stocks and the Cox-Ingersoll-Ross model on the spot interest rate are simple examples. Then we shall give a series of examples on the valuation formulae including plain vanilla options, average options, and other contingent claims. We shall also give some numerical evidence of the accuracy of the approximations we have obtained for practical purposes. Our approach can be rigorously justified by an infinite dimensional

mathematics, the Malliavin-Watanabe-Yoshida theory recently developed in stochastic analysis.

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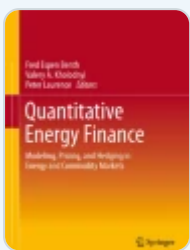
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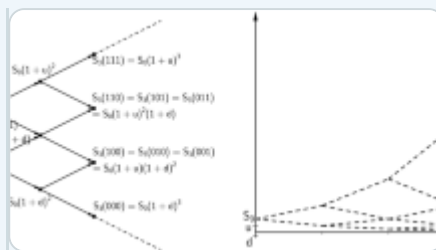
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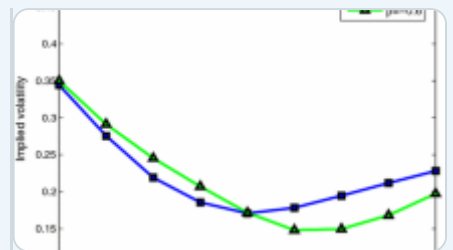
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References

Fujikoshi, Y., Morimune, K., Kunitomo, N., and Taniguchi, M. (1982) Asymptotic

expansions of the distributions of the estimates of coefficients in a simultaneous equation system, *J. Econometrics* **18**, 191-205.

[Google Scholar](#)

He, Hua (1992) Option prices with stochastic volatilities: An equilibrium analysis, working paper, University of California, Berkeley.

[Google Scholar](#)

He, H. and Takahashi, A. (1996) A variable reduction technique for pricing average-rate options, *Jap. J. Financ. Econom.* **1**, 3-23.

[Google Scholar](#)

Heath, D., Jarrow, R., and Morton, A. (1992) Bond pricing and the term structure of interest rates: A new methodology for contingent claims valuation, *Econometrica* **60**, 77-105.

[Google Scholar](#)

Ikeda, N. and Watanabe, S. (1989) *Stochastic Differential Equations and Diffusion Processes*, 2nd edn, North-Holland/Kodansha, Tokyo.

[Google Scholar](#)

Kunitomo, N. and Takahashi, A. (1992) Pricing average options, *Jap. Financ. Rev.* **14**, 1-20 (in Japanese).

[Google Scholar](#)

Kunitomo, N. and Takahashi, A. (1995) The asymptotic expansion approach to the valuation of interest rate contingent claims, Discussion Paper No. 95-F-19, Faculty of Economics, University of Tokyo.

Kunitomo, N. and Takahashi, A. (1998) On validity of the asymptotic expansion approach in contingent claim analysis, Discussion Paper No. 98-F-6, Faculty of Economics, University of Tokyo.

Kusuoka, S. and Stroock, D. (1982) Applications of the Malliavin calculus, Part I, Taniguchi Symp. SA Katata, pp. 271-306.

Takahashi, A. (1995) Essays on the valuation problems of contingent claims, Unpublished PhD Dissertation, University of California, Berkeley.

[Google Scholar](#)

Watanabe, S. (1984) *Lectures on Stochastic Differential Equations and Malliavin Calculus*, Tata Institute of Fundamental Research, Springer-Verlag.

[Google Scholar](#)

Watanabe, S. (1987) Analysis of Wiener functionals (Malliavin Calculus) and its applications to heat kernels, *The Annals of Probability* **15**, 1-39.

[Google Scholar](#)

Yoshida, N. (1992a) Asymptotic expansions for statistics related to small diffusions, *J. Jap. Statist. Soc.* **22**, 139-159.

[Google Scholar](#)

Yoshida, N. (1992b) Asymptotic expansions of maximum likelihood estimators for small diffusions via the theory of Malliavin-Watanabe, *Probability Theory and Related Fields* **92**, 275-311.

[Google Scholar](#)

Yoshida, N. (1997) Malliavin calculus and asymptotic expansions for martingales,

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Cite this article

Takahashi, A. An Asymptotic Expansion Approach to Pricing Financial Contingent Claims. *Asia-Pacific Financial Markets* **6**, 115–151 (1999). <https://doi.org/10.1023/A:1010080610650>

Issue Date

June 1999

DOI

<https://doi.org/10.1023/A:1010080610650>

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