Copula–Based Models for Financial Time Series

Chapter | First Online: 01 January 2009

pp 767–785 | Cite this chapter

Handbook of Financial Time Series

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Abstract

This paper presents an overview of the literature on applications of copulas in the modelling of financial time series. Copulas have been used both in multivariate time series analysis, where they are used to characterize the (conditional) cross-sectional dependence between individual time series, and in univariate time series analysis, where they are used to characterize the dependence between a sequence of observations of a scalar time series process. The paper includes a broad, brief, review of the many applications of copulas in finance and economics.

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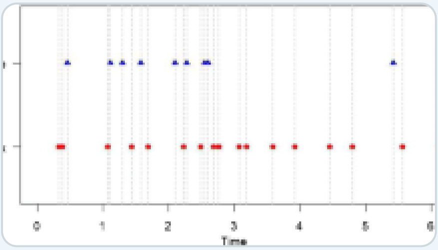

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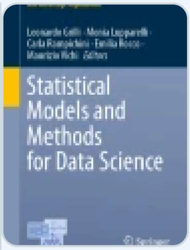

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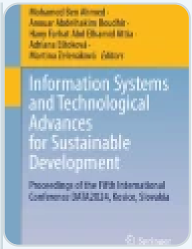

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About this chapter

Cite this chapter

Patton, A.J. (2009). Copula–Based Models for Financial Time Series. In: Mikosch, T., Kreiß, JP., Davis, R., Andersen, T. (eds) Handbook of Financial Time Series. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-71297-8_34

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DOI
https://doi.org/10.1007/978-3-540-71297-8_34

Published
10 February 2009

Publisher Name
Springer, Berlin, Heidelberg

Print ISBN
978-3-540-71296-1

Online ISBN
978-3-540-71297-8

eBook Packages
Mathematics and Statistics
Mathematics and Statistics (R0)

Keywords

Marginal Distribution | Asset Return | Tail Dependence | Multivariate Time Series | Copula Model

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