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Copula–Based Models for Financial Time Series

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Abstract

This paper presents an overview of the literature on applications of copulas in the modelling of financial time series. Copulas have been used both in multivariate time series analysis, where they are used to characterize the (conditional) cross-sectional dependence between individual time series, and in univariate time series analysis, where they are used to characterize the dependence between a sequence of observations of a scalar time series process. The paper includes a broad, brief, review of the many applications of copulas in finance and economics.

Keywords

Marginal Distribution

Asset Return

These keywords were added by machine and not by the authors. This process is experimental and the keywords may be updated as the learning algorithm improves.

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