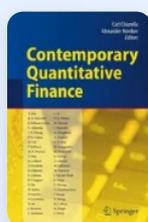


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# The Economic Plausibility of Strict Local Martingales in Financial Modelling

| Chapter

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## Contemporary Quantitative Finance

[Hardy Hulley](#) 

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## Abstract

The context for this article is a continuous financial market consisting of a risk-free savings account and a single non-dividend-paying risky security. We present two concrete models for this market, in which strict local martingales play decisive roles. The first admits an equivalent risk-neutral probability measure under which the discounted price of the risky security is a strict local martingale, while the second model does not even admit an equivalent risk-neutral probability measure, since the putative density process for such a measure is itself a strict local martingale. We highlight a number of apparent anomalies associated with both models that may offend the sensibilities of the classically-educated reader. However, we also demonstrate that these issues are easily resolved if one thinks

economically about the models in the right way. In particular, we argue that there is nothing inherently objectionable about either model.

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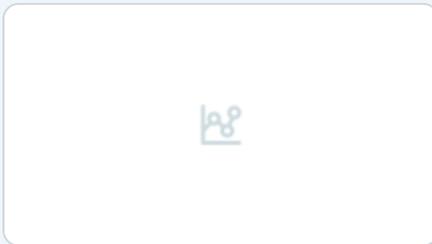
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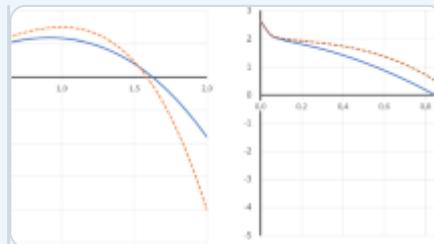
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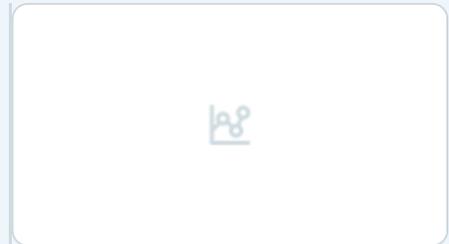
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*For Eckhard Platen on the occasion of his 60th birthday, with gratitude.*

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