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# Higher degree stop-loss transforms and stochastic orders — (I) Theory

Stop-Loss Transformierte eines höheren Grades und stochastische Ordnungen - (I) Theorie

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## Zusammenfassung

Die Stop-Loss Transformierten eines höheren Grades und ihre logarithmische Ableitungen, genannt Stop-Loss Raten eines höheren Grades, werden untersucht, um Einsicht in die Hierarchie der Stop-Loss Ordnungen und verwandte stochastische Ordnungen zu erlangen. Mit Hilfe von Differential und Integral rekursive Relationen werden zwei Charakterisierungen von Gupta und Gupta (1983) auf einfache Weise hergeleitet. Diese Resultate zeigen, daß eine Verteilungsfunktion eindeutig durch eine Stop-Loss Transformierte oder Stop-Loss Rate eines höheren Grades definiert ist. Klassen ISLR (n) von Verteilungen mit einer wachsenden Stop-Loss Rate des Grades n werden betrachtet. Es wird gezeigt, daß die Eigenschaft ISLR (n) die Eigenschaft ISLR (n+1) zur Folge hat, was das wohlbekannte Resultat von Bryson and Siddiqui (1969) für den Fall n=0

verallgemeinert. Hinreichende Bedingungen für eine Stop-Loss Ordnung eines höheren Grades werden anhand von Stop-Loss Raten Ordnungen und Stop-Loss Raten Gefährlichkeitsordnungen formuliert. Zwei neue Charakterisierungen der Stop-Loss Ordnungen eines höheren Grades, welche die Vorzeichenänderungen der Stop-Loss Transformierten und der Stop-Loss Raten berücksichtigt, werden aufgestellt. Anwendungen in Versicherungsmathematik folgen in Teil (II) dieser Abhandlung.

## Summary

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The higher degree stop-loss transforms and their logarithmic derivatives, called higher degree stop-loss rate functions, are studied to get insight into the hierarchical theory of the higher degree stop-loss orders and related stochastic orders. Based on differential-integral recursive relationships, we derive in a simple way two characterization results by Gupta and Gupta (1983), which state that the higher degree stop-loss transforms and the higher degree stop-loss rate functions uniquely determine a distribution function. Classes ISLR ( $n$ ) of distributions with an increasing stop-loss rate function of degree  $n$  are considered, and it is shown that ISLR ( $n$ ) implies ISLR ( $n+1$ ). This result generalizes the well-known fact by Bryson and Siddiqui (1969) that a distribution with an increasing failure rate has necessarily a decreasing mean residual life. Necessary and sufficient conditions, which guarantee that ISLR ( $n+1$ ) implies ISLR ( $n$ ), are formulated. Using notions of higher degree stop-loss rate order and higher degree stop-loss rate dangerousness order, sufficient conditions for a higher degree stop-loss order relation are established. Two new sign change characterizations of the higher degree stop-loss order by means of higher degree stop-loss transforms and higher degree stop-loss rate functions are derived. Applications in actuarial mathematics follow in part (II) of the present work.



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