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Robust portfolios: contributions from operations research and finance

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Abstract

In this paper we provide a survey of recent contributions to robust portfolio strategies from operations research and finance to the theory of portfolio selection. Our survey covers results derived not only in terms of the standard mean-variance objective, but also in terms of two of the most popular risk measures, mean-VaR and mean-CVaR developed recently. In addition, we review optimal estimation methods and Bayesian robust approaches.

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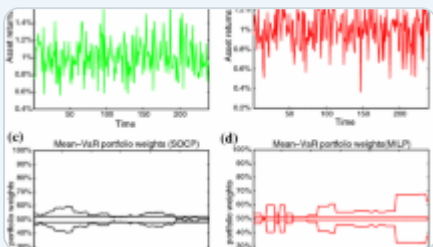
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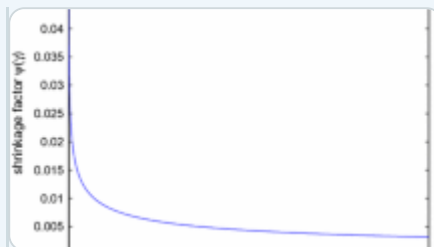
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