

[Home](#) > [Annals of Operations Research](#) > [Article](#)

Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities

| Original Paper | Published: 05 April 2016

| Volume 248, pages 253–280, (2017) [Cite this article](#)



[Annals of Operations Research](#)

[Aims and scope](#) →

[Submit manuscript](#) →

[Chandra K. Jaggi](#)  orcid.org/0000-0001-6179-8376¹, [Sunil Tiwari](#)¹ & [Satish K. Goel](#)¹

 783 Accesses  82 Citations [Explore all metrics](#) →

Abstract

The formulation of classical deteriorating inventory models is done with the common unrealistic assumption that all the items start deteriorating as soon as they arrive in the warehouse. On the contrary, in a realistic environment, it has been observed that there are several items that do not deteriorate immediately. Items such as dry fruits, potatoes, yams and even some fruits and vegetables have a shelf life and deteriorate only after a time lag. Apart from this, in today's competitive business world, the supplier usually offers a trade credit period to his retailers to attract more sales and the retailer considers it as an alternative to price discount. The present research proposes a two warehouse inventory model for non-instantaneous deteriorating items under trade credit based on the above

phenomena, where the demand rate is assumed to be a function of the selling price. Further, shortages are completely backlogged and the interest on shortages at the beginning of the cycle has also been considered. The objective of the study is to determine the retailer's optimal replenishment policies that maximize the average profit per unit time. Conclusively, a numerical example is presented to illustrate the applicability of the proposed model. Sensitivity analysis on key parameters is provided to reveal the managerial insights.

 This is a preview of subscription content, [log in via an institution](#)  to check access.

Access this article

[Log in via an institution](#) →

[Buy article PDF 39,95 €](#)

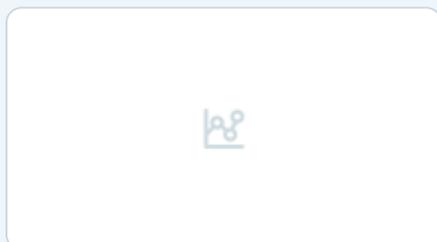
Price includes VAT (Poland)

Instant access to the full article PDF.

Rent this article via [DeepDyve](#) 

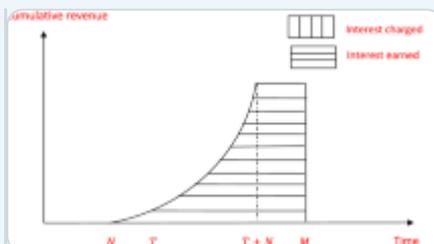
[Institutional subscriptions](#) →

Similar content being viewed by others



[Partial Trade Credit Policy of Retailer in Economic Order Quantity Models for Deteriorating Items with...](#)

Article | 19 April 2015



[Optimal inventory policies for deteriorating items with expiration date and dynamic demand under two-level trad...](#)

Article | 06 February 2021



[The Inventory Model for Deteriorating Items with Permissible Delay in Payment and Investment in Preservati...](#)

Article | 19 October 2023

References

Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *Journal of the Operational Research Society*, 46(5), 658-662.

[Article](#) [Google Scholar](#)

Arcelus, F. J., & Srinivasan, G. (1993). Delay of payments for extraordinary purchases. *Journal of the Operational Research Society*, 44(8), 785-795.

[Article](#) [Google Scholar](#)

Bakker, M., Riezebos, J., & Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, 221(2), 275-284.

[Article](#) [Google Scholar](#)

Chang, C. T., Teng, J. T., & Goyal, S. K. (2010). Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. *International Journal of Production Economic*, 123(1), 62-68.

[Article](#) [Google Scholar](#)

Chen, S.-C., Cárdenas-Barrón, L. E., & Teng, J. T. (2014). Retailers economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. *International Journal of Production Economics*, 155, 284-291.

[Article](#) [Google Scholar](#)

Chung, K. J. (1998). A theorem on the determination of economic order quantity under conditions of permissible delay in payments. *Computer and Operations*

[Article](#) [Google Scholar](#)

Chung, K. J., & Liao, J. J. (2004). Lot-sizing decisions under trade credit depending on the ordering quantity. *Computers and Operations Research*, 31(6), 909-928.

[Article](#) [Google Scholar](#)

Chung, K. J., & Liao, J. J. (2006). The optimal ordering policy in a DCF analysis for deteriorating items under trade credit depending on the ordering quantity. *International Journal of Production Economics*, 100(1), 116-130.

[Article](#) [Google Scholar](#)

Chung, K. J., & Liao, J. J. (2009). The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. *European Journal of Operational Research*, 196(2), 563-568.

[Article](#) [Google Scholar](#)

Chung, K. J., & Liao, J. J. (2011). The simplified solution algorithm for an integrated supplier-buyer inventory model with two-part trade credit in a supply chain system. *European Journal of Operational Research*, 213(1), 156-165.

[Article](#) [Google Scholar](#)

Chung, K. J., & Cárdenas-Barrón, L. E. (2013). The simplified solution procedure for deteriorating items under stock-dependent demand and two-level trade-credit in the supply chain management. *Applied Mathematical Modelling*, 37(7), 4653-4660.

[Article](#) [Google Scholar](#)

Chung, K. J., Cárdenas-Barrón, L. E., & Ting, P. S. (2014). An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade-credit. *International Journal of Production Economics*, 155, 310–317.

[Article](#) [Google Scholar](#)

Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. *American Institute of Industrial Engineering Transactions*, 5(4), 323–326.

[Google Scholar](#)

Dye, C. Y. (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega*, 41(5), 872–880.

[Article](#) [Google Scholar](#)

Geetha, K. V., & Uthayakumar, R. (2010). Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. *Journal of Computational and Applied Mathematics*, 233(10), 2492–2505.

[Article](#) [Google Scholar](#)

Ghare, P. M., & Shrader, G. F. (1963). A model for exponentially decaying inventories. *Journal of Industrial Engineering*, 14(5), 238–243.

[Google Scholar](#)

Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, 36(4), 335–338.

[Article](#) [Google Scholar](#)

Goyal, S. K., & Giri, B. C. (2001). Recent trends in modelling of deteriorating

inventory. *European Journal of Operational Research*, 134(1), 1–16.

[Article](#) [Google Scholar](#)

Haley, C. W., & Higgins, R. C. (1973). Inventory policy and trade credit financing. *Management Science*, 20(4), 464–471.

[Article](#) [Google Scholar](#)

Hartely, V. R. (1976). *Operations research: A managerial emphasis*. California: Good Year Publishing Company.

[Google Scholar](#)

Hsieh, T. P., Dye, C. Y., & Ouyang, L. Y. (2008). Determining optimal lot size for a two-warehouse system with deterioration and shortages using net present value. *European Journal of Operational Research*, 191(1), 182–192.

[Article](#) [Google Scholar](#)

Hwang, H., & Shinn, S. W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operations Research*, 24(6), 539–547.

[Article](#) [Google Scholar](#)

Huang, K. N., & Liao, J. J. (2008). A simple method to locate the optimal solution for exponentially deteriorating items under trade credit financing. *Computers and Mathematics with Applications*, 56(4), 965–977.

[Article](#) [Google Scholar](#)

Jaggi, C. K., & Aggarwal, S. P. (1994). Credit financing in economic ordering policies of deteriorating items. *International Journal of Production Economics*, 34(2), 151–155.

[Article](#) [Google Scholar](#)

Jaggi, C. K., & Verma, P. (2010). An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. *International Journal of Services Operations and Informatics*, 5(3), 209–230.

[Article](#) [Google Scholar](#)

Jaggi, C. K., & Tiwari, S. (2014). Two-Warehouse inventory model for non-instantaneous deteriorating items with price dependent demand and time-varying holding cost. In Om Parkash (Ed.), *Mathematical modeling and applications* (pp. 225–238). Germany: LAMBERT Academic Publishers.

[Google Scholar](#)

Jaggi, C. K., Sharma, A., & Tiwari, S. (2015a). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand under permissible delay in payments: A new approach. *International Journal of Industrial Engineering Computations*, 6(4), 481–502.

[Article](#) [Google Scholar](#)

Jaggi, C. K., Tiwari, S., & Shafi, A. (2015b). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers and Industrial Engineering*, 88, 378–385.

[Article](#) [Google Scholar](#)

Jamal, A. M. M., Sarker, B. R., & Wang, S. (1997). An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *Journal of the Operational Research Society*, 48(8), 826–833.

[Article](#) [Google Scholar](#)

Jamal, A. M. M., Sarker, B. R., & Wang, S. (2000). Optimal payment time for a

retailer under permitted delay of payment by the wholesaler. *International Journal of Production Economics*, 66(1), 59–66.

[Article](#) [Google Scholar](#)

Lee, C. C. (2006). Two-warehouse inventory model with deterioration under FIFO dispatching policy. *European Journal of Operational Research*, 174(2), 861–873.

[Article](#) [Google Scholar](#)

Lee, C. C., & Hsu, S. L. (2009). A two-warehouse production model for deteriorating inventory items with time-dependent demands. *European Journal of Operational Research*, 194(3), 700–710.

[Article](#) [Google Scholar](#)

Li, Y., Zhen, X., & Cai, X. (2014). Trade credit insurance, capital constraint, and the behavior of manufacturers and banks. *Annals of Operations Research*, 1–20. doi:[10.1007/s10479-014-1602-x](https://doi.org/10.1007/s10479-014-1602-x).

Liang, Y., & Zhou, F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. *Applied Mathematical Modelling*, 35(5), 2221–2231.

[Article](#) [Google Scholar](#)

Liao, J. J., & Chung, K. J. (2009). An EOQ model for deterioration items under trade credit policy in a supply chain system. *Journal of the Operations Research Society of Japan*, 52(1), 46.

[Google Scholar](#)

Liao, J. J., Huang, K. N., & Chung, K. J. (2012). Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit. *International Journal of Production Economics*, 137(1), 102–115.

Liao, J. J., Huang, K. N., & Chung, K. J. (2013). Optimal pricing and ordering policy for perishable items with limited storage capacity and partial trade credit. *IMA Journal of Management Mathematics*, 24(1), 45-61.

[Article](#) [Google Scholar](#)

Mahata, G. C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert Systems with Applications*, 39(3), 3537-3550.

[Article](#) [Google Scholar](#)

Maihami, R., & Kamalabadi, I. N. (2012a). Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demands. *International Journal of Production Economics*, 136(1), 116-122.

[Article](#) [Google Scholar](#)

Maihami, R., & Kamalabadi, I. N. (2012b). Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. *Mathematical and Computer Modelling*, 55(5-6), 1722-1733.

[Article](#) [Google Scholar](#)

Niu, B., & Xie, J. (2008). A note on Two-warehouse inventory model with deterioration under FIFO dispatches policy. *European Journal of Operational Research*, 190(2), 571-577.

[Article](#) [Google Scholar](#)

Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2006). A study on an inventory model for

non-instantaneous deteriorating items with permissible delay in payments.

Computers and Industrial Engineering, 51(4), 637-651.

[Article](#) [Google Scholar](#)

Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2008). Retailer's ordering policy for non-instantaneous deteriorating items with quantity discount, stock dependent demand and stochastic backorder rate. *Journal of the Chinese Institute of Industrial Engineers*, 25(1), 62-72.

[Article](#) [Google Scholar](#)

Ouyang, L. Y., Yang, C. T., Chan, Y. L., & Cárdenas-Barrón, L. E. (2013). A comprehensive extension of the optimal replenishment decisions under two-level of trade-credit policy depending on the order quantity. *Applied Mathematics and Computation*, 224, 268-277.

[Article](#) [Google Scholar](#)

Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, 42(1), 27-37.

[Article](#) [Google Scholar](#)

Sarker, B. R., Jamal, A. M. M., & Wang, S. (2001). Optimal payment time under permissible delay in payment for products with deterioration. *Production Planning and Control*, 11(4), 380-390.

[Article](#) [Google Scholar](#)

Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2014). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research*, 229(1), 677-702.

[Article](#) [Google Scholar](#)

Shah, N. H., Soni, H. N., & Patel, K. A. (2013). Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega*, 41(2), 421–430.

[Article](#) [Google Scholar](#)

Soni, H., & Patel, K. (2012). Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: Fuzzy expected value model. *International Journal of Industrial Engineering Computations*, 3(3), 281–300.

[Article](#) [Google Scholar](#)

Thangam, A. (2012). Optimal price discounting and lot-sizing policies for perishable items in a supply chain under advance payment scheme and two-echelon trade credits. *International Journal of Production Economics*, 139(2), 459–472.

[Article](#) [Google Scholar](#)

Wu, K. S., Ouyang, L. Y., & Yang, C. T. (2006). An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics*, 101(2), 369–384.

[Article](#) [Google Scholar](#)

Wu, K. S., Ouyang, L. Y., & Yang, C. T. (2009). Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand. *International Journal of System Sciences*, 40(12), 1273–1281.

[Article](#) [Google Scholar](#)

Wu, J., Ouyang, L. Y., Cárdenas-Barrón, L. E., & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-

level trade-credit financing. *European Journal of Operational Research*, 237(3), 898–908.

[Article](#) [Google Scholar](#)

Yang, H. L. (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation. *European Journal of Operational Research*, 157(1), 344–356.

[Article](#) [Google Scholar](#)

Yang, H. L. (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production Economics*, 103(1), 362–370.

[Article](#) [Google Scholar](#)

Yang, H. L. (2012). Two-warehouse partial backlogging inventory models with three parameters Weibull distribution deterioration under inflation. *International Journal of Production Economics*, 138(1), 107–116.

[Article](#) [Google Scholar](#)

Zhou, Y. W., & Yang, S. L. (2005). A two-warehouse inventory model for items with stock-level dependent demand rate. *International Journal of Production Economics*, 95(2), 215–228.

[Article](#) [Google Scholar](#)

Zhong, Y. G., & Zhou, Y. W. (2013). Improving the supply chain's performance through trade credit under inventory-dependent demand and limited storage capacity. *International Journal of Production Economics*, 143(2), 364–370.

[Article](#) [Google Scholar](#)

Acknowledgments

The authors would like to thank the Editor Endre Boros and anonymous referee for their valuable and constructive comments on earlier versions of our paper, which have led to a significant improvement in the manuscript. The first author acknowledges the support of the University Grants Commission through University of Delhi (Research Grant No. RC/2015/9677). The second author would like to thank University Grant Commission (UGC) for providing the Non-NET fellowship to accomplish this research.

Author information

Authors and Affiliations

**Department of Operational Research, Faculty of Mathematical Sciences,
New Academic Block, University of Delhi, Delhi, 110007, India**

Chandra K. Jaggi, Sunil Tiwari & Satish K. Goel

Corresponding author

Correspondence to [Chandra K. Jaggi](#).

Appendices

Appendix 1

For Case 1, the necessary conditions for maximizing the total average profit is given by

$$\begin{aligned} \frac{\partial \text{TP}_{1.1}}{\partial t_r} = & \frac{1}{T} \left[\frac{D}{p} \left(\beta \left(cI_p + F \right) \left(1 - e^{-\beta} \right)^{\left(t_r - t_d \right)} \right) \right. \\ & \left. + \left(cI_p - H \right) \left(W e^{-\alpha \left(t_r - t_d \right)} + \frac{D}{p} \right) \right. \\ & \left. \left(1 - e^{-\alpha \left(t_w - t_r \right)} \right) \left(1 - X_1 \right) \right] \\ & - \frac{D}{p} t_d e^{-\beta \left(t_r - t_d \right)} + s D \left(p \right) \left(T - t_w \right) X_1 - \end{aligned}$$

$$\begin{aligned}
& cD\left(p \right) \left(e^{\beta \left(t_r - t_d \right)} + X_1 \right) \right. \\
& \left. - pI_e D\left(p \right) MX_1 - cI_p D\left(p \right) e^{\beta \left(t_r - t_d \right)} \left(t_d - M \right) \right] = 0 \quad \frac{\partial \text{TP}_{1.1}}{\partial T} = \frac{1}{T^2} \left[D\left(p \right) pT \right. \\
& \left. - A - F \left(Z t_d - \frac{D\left(p \right) t_d^2}{2} - W t_d - \frac{D\left(p \right)}{\beta} \left(t_r - t_d + \frac{1}{\beta} - \frac{e^{\beta \left(t_r - t_d \right)}}{\beta} \right) \right) \right. \\
& \left. + H \left(W t_d + \frac{W}{\alpha} \left(1 - e^{\alpha \left(t_d - t_r \right)} \right) + \frac{D\left(p \right)}{\alpha} \left(t_r - t_w - \frac{1}{\alpha} + \frac{e^{\alpha \left(t_w - t_r \right)}}{\alpha} \right) \right) \right. \\
& \left. - sD\left(p \right) \left(T - t_w \right)^2 / 2 - c \left(W + D\left(p \right) t_d + \frac{D\left(p \right)}{\beta} \left(e^{\beta \left(t_r - t_d \right)} - 1 \right) - D\left(p \right) \left(T - t_w \right) \right) \right. \\
& \left. + \frac{\left(pI_e D\left(p \right) M^2 \right)}{2} + pI_e D\left(p \right) \left(T - t_w \right) M \right] \quad - cI_p \left(Z \left(t_d - M \right) + \frac{W}{\alpha} \left(1 - e^{\alpha \left(t_d - t_r \right)} \right) \right. \\
& \left. + \frac{D\left(p \right)}{\alpha^2} \left(e^{\alpha \left(t_w - t_r \right)} - 1 \right) \right) \right. \\
& \left. + \frac{D\left(p \right)}{\alpha} \left(t_w - t_r \right) - \frac{D\left(p \right)}{2} \left(t_d^2 - M^2 \right) \right) \right. \\
& \left. + \frac{D\left(p \right)}{\beta^2} \left(e^{\beta \left(t_r - t_d \right)} - 1 \right) - \frac{D\left(p \right)}{\beta} \left(t_r - t_d \right) \right] + \frac{1}{T} \left[D\left(p \right) p - sD\left(p \right) \left(T - t_w \right) \right] \right. \\
& \left. + 2cD\left(p \right) + pI_e D\left(p \right) M \right] = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{1.1}}{\partial p} = \frac{1}{T} \left[\left(1 - e \right) D\left(p \right) T + F \left(\frac{t_d^2}{2} + \frac{1}{\beta} \left(t_r - t_d + \frac{1}{\beta} \right) \left(1 - e^{\beta \left(t_r - t_d \right)} \right) \right) \right. \\
& \left. + H \left(\frac{t_r - t_w - \frac{1}{\alpha} \left(1 - e^{\alpha \left(t_w - t_r \right)} \right)}{\alpha} + \frac{HD\left(p \right)}{\alpha} \left(e^{\alpha \left(t_w - t_r \right)} - 1 \right) + s \left(T - t_w \right) \right) \right. \\
& \left. - \frac{D\left(p \right) X_3 - \frac{\left(T - t_w \right)^2 X_2}{2}}{X_2} - c \left(t_d + \frac{1}{\beta} \left(e^{\beta \left(t_r - t_d \right)} - 1 \right) - \left(T - t_w \right) \right) \right] \quad - cD\left(p \right)
\end{aligned}$$

$$\begin{aligned}
& +\frac{D\left(p\right) I_e M^{\{2\}}}{\{2\}} \nonumber \&\left. +\frac{D\left(p\right) I_e}{\left(\left\{T-t_w\right\}\right) M-c I_p} \left\{\frac{1}{\alpha^{\{2\}}}\right\} \left(\left\{e^{\{\alpha\left(\left\{t_w-t_r\right\}\right)-1}\right\}}\right) \right\} \\
& X_2 \right. \nonumber \&\left. -\frac{1}{\alpha} \left(\left\{t_w-t_r\right\}\right) \right\} \\
& \left. X_3 \right] =0 \end{aligned}$$

(38)

$$\begin{aligned}
& \frac{\partial \textit{TP}_{\{1.4\}}}{\partial t_r} = \&\left\{ \frac{1}{T} \left[\frac{FD\left(p\right)}{\beta} \left(\left\{1-e^{\{\beta\left(\left\{t_r-t_d\right\}\right)}\right\}\right) \right] - H \left\{ \left\{W e^{-\alpha\left(\left\{t_r-t_d\right\}\right)} + \frac{D\left(p\right)}{\alpha} \left\{ \left(\left\{1-e^{\{\alpha\left(\left\{t_w-t_r\right\}\right)}\right\}\right) \right\} \right) \right\} \right. \\
& \left. \left\{ \left(1-X_1\right) \right\} \right\} \right\} \right. \&\left. \left\{ -\frac{FD\left(p\right) t_d}{e^{\{\beta\left(\left\{t_r-t_d\right\}\right)} + s D\left(p\right) \left(\left\{T-t_w\right\}\right) X_1 - c D\left(p\right) \left(\left\{e^{\{\beta\left(\left\{t_r-t_d\right\}\right)} + X_1\right\}\right) + p I_e D\left(p\right) \left(\left\{3 t_w-2 M\right\}\right) X_1} \right) \right\} \right] =0 \\
& \frac{\partial \textit{TP}_{\{1.4\}}}{\partial T} = \&\left\{ -\frac{1}{T^{\{2\}}} \left[\frac{D\left(p\right) p T-A-F}{\left\{Z t_d-\frac{D\left(p\right) t_d^2}{2}-W t_d-\frac{D\left(p\right)}{\beta} \left(\left\{t_r-t_d+\frac{1}{\beta}\right\}-\frac{e^{\{\beta\left(\left\{t_r-t_d\right\}\right)}}{\beta}\right\}\right) \right\} \right. \\
& \left. \left\{ \left(\beta\right) \right\} \right\} \right\} \right. \&\left. -\frac{H}{\left\{W t_d+\frac{W}{\alpha} \left(\left\{1-e^{\{\alpha\left(\left\{t_d-t_r\right\}\right)}\right\}\right) + \frac{D\left(p\right)}{\alpha} \left(\left\{t_r-t_w-\frac{1}{\alpha}\right\}+\frac{e^{\{\alpha\left(\left\{t_w-t_r\right\}\right)}}{\alpha}\right\}\right) \right\} \right\} \right. \\
& \left. -s D\left(p\right) \left(\left\{T-t_w\right\}\right)^{\{2\}} / 2 \&- \frac{c}{\left\{W+D\left(p\right) t_d+\frac{D\left(p\right)}{\beta} \left(\left\{e^{\{\beta\left(\left\{t_r-t_d\right\}\right)}-1\right\}\right) -D\left(p\right) \left(\left\{T-t_w\right\}\right) \right\} \right\} \right. \\
& \left. \&\left. \left\{ +\frac{p I_e D\left(p\right) t_w^2}{2}-p I_e D\left(p\right) t_w \left(\left\{M-t_w\right\}\right) +p I_e D\left(p\right) \left(\left\{T-t_w\right\}\right) M \right\} \right] \&+\frac{1}{T} \left[\frac{D\left(p\right) p-s D\left(p\right) \left(\left\{T-t_w\right\}\right) +2 c D\left(p\right) +p I_e D\left(p\right) M}{\left(\left\{T-t_w\right\}\right) +2 c D\left(p\right) +p I_e D\left(p\right) M} \right] =0 \\
& \end{aligned}$$

$$\begin{aligned}
& \frac{\partial \textit{TP}_{\{1.4\}}}{\partial p} = \&\left\{ \frac{1}{T} \left[\left(\left\{1-e\right\}\right) D\left(p\right) T+F \left\{ \frac{t_d^2}{2} \right\} \right. \\
& \left. \left\{ 2+\frac{1}{\beta} \left(\left\{t_r-t_d+\frac{1}{\beta}\right\}\right) \left(\left\{1-e^{\{\beta\left(\left\{t_r-t_d\right\}\right)}\right\}\right) \right\} \right\} \right. \\
& \left. X_2 \right. \nonumber \&\left. -\frac{H}{\left(\left\{T-t_w\right\}\right) +2 c D\left(p\right) +p I_e D\left(p\right) M} \right] =0 \\
& \end{aligned}$$

$$\begin{aligned}
& \left\{ \frac{1}{\alpha} \left(1 - e^{-\alpha(t_w - t_r)} \right) \right\} \left(X_2 - \frac{HD}{\alpha} \left(p \right) \right) \left(e^{-\alpha(t_w - t_r)} - 1 \right) X_3 + s \left(T - t_w \right) \\
& \left(D \left(p \right) X_3 - \frac{\left(T - t_w \right)^2 X_2}{2} \right) \\
& \left(t_d + \frac{1}{\beta} \left(e^{-\beta(t_r - t_d)} - 1 \right) - \left(T - t_w \right) \right) X_2 - \frac{D \left(p \right) X_3 + \left(\frac{p I_e M^2}{2} + p I_e \left(T - t_w \right) M \right)}{2} \\
& \left(I_e D \left(p \right) t_w^2 \right) / 2 + p I_e D \left(p \right) t_w X_3 \right. \\
& \left. - I_e D \left(p \right) t_w \left(M - t_w \right) \right. \\
& \left. - p I_e D \left(p \right) \left(M - t_w \right) X_3 - F t_d \left(t_d + \frac{1}{\beta} \left(e^{-\beta(t_r - t_d)} - 1 \right) \right) \right. \\
& \left. + p I_e D \left(p \right) t_w X_3 \right. \\
& \left. - I_e D \left(p \right) \left(T - t_w \right) M - p I_e D \left(p \right) M X_3 \right) = 0
\end{aligned}$$

(39)

Appendix 2

For Case 2, the necessary conditions for maximizing the total average profit is given by

$$\begin{aligned}
\frac{\partial TP_{2.1}}{\partial t_r} &= \frac{D \left(p \right) \left\{ T \left[F t_r + H \left(t_d - t_r \right) + \frac{1}{\alpha} \left(1 - e^{-\alpha(t_w - t_d)} \right) \right] Y_1 \right\}}{s \left(T - t_w \right) Y_1 + c \left(e^{-\beta(t_r - t_d)} + Y_1 \right) + c \left(1 + Y_1 \right)} - \frac{p I_e M Y_1 + c I_p \left(t_d - t_r \right) + \frac{1}{\alpha} \left(1 - e^{-\alpha(t_w - t_d)} \right) Y_1}{\left(t_r - M \right)} - \frac{HW}{T} = 0 \\
\frac{\partial TP_{2.1}}{\partial T} &= - \frac{1}{T^2} \left[D \left(p \right) p T A - \frac{FD \left(p \right) t_r^2}{2} \right] - H \left(W t_r + D \left(p \right) t_r \left(t_d - t_r \right) - \frac{D \left(p \right)^2 \left(t_d^2 - t_r^2 \right)}{2} \right) \\
&+ \frac{D \left(p \right)}{\alpha} \left(\frac{1}{\alpha} \left(e^{-\alpha(t_w - t_d)} - 1 \right) - \left(t_w - t_d \right) \right) - \frac{D \left(p \right) \left(T - t_w \right)^2}{2} - c \left(W + D \left(p \right) t_r - D \left(p \right) \left(T - t_w \right) \right)
\end{aligned}$$

$$\begin{aligned} & \left(\frac{1}{2} + p I_e D \left(p \right) M^2 \right) \left(T - t_w \right) M - c I_p \left(W \left(t_d - M \right) + D \left(p \right) t_r \left(t_d - t_r \right) \right) \\ & - \frac{D \left(p \right) \left(t_d - t_r \right)^2}{\alpha^2} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \frac{D \left(p \right)}{\alpha} \left(t_w - t_d \right) + \frac{D \left(p \right)^2 \left(t_r - M \right)^2}{\alpha} \\ & + \frac{1}{T} \left(D \left(p \right) p - s D \left(p \right) \left(T - t_w \right) + 2 c D \left(p \right) + p I_e D \left(p \right) M \right) = 0 \end{aligned}$$

$$\begin{aligned} & \frac{\partial \text{TP}_{2.1}}{\partial p} = \frac{1}{T} \left[\left(1 - e \right) D \left(p \right) T - \frac{F Y_2 t_r^2}{2} \right] - H Y_2 \left[t_r \left(t_d - t_r \right) - \frac{1}{2} \left(t_d^2 - t_r^2 \right) \right] \\ & + \frac{1}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right) - H \frac{D \left(p \right)}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) \\ & - Y_3 \left[s Y_2 \left(T - t_w \right)^2 / 2 + s D \left(p \right) \left(T - t_w \right) Y_3 - c Y_2 \left(t_r - \left(T - t_w \right) \right) - c D \left(p \right) Y_3 - p I_e D \left(p \right) M Y_3 \right] \\ & + \frac{Y_2}{\alpha^2} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) \left[\frac{D \left(p \right)}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) Y_3 - \frac{Y_2}{2} \left(t_r - M \right)^2 \right] = 0 \end{aligned}$$

(40)

$$\begin{aligned} & \frac{\partial \text{TP}_{2.2}}{\partial t_r} = \frac{-D \left(p \right) T}{\alpha} \left[F t_r + H \left(t_d - t_r \right) + \frac{1}{\alpha} \left(1 - e^{\alpha \left(t_w - t_d \right)} \right) Y_1 \right] \\ & - s \left(T - t_w \right) Y_1 - c \left(e^{\beta \left(t_r - t_d \right)} + Y_1 \right) + c \left(1 + Y_1 \right) - p I_e M Y_1 + c I_p \left(t_d - M \right) + \frac{1}{\alpha} \left(1 - e^{\alpha \left(t_w - t_d \right)} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.2}}{\partial T} = \frac{1}{T^2} \left[D(p) p T - A \frac{FD(p)}{t_r^2} \right] - H \left[W t_r + D(p) t_r \left(\frac{t_d - t_r}{t_r} \right) \right] \\
& - \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) - \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& - s D(p) \left(\frac{T - t_w}{t_r} \right)^2 / 2 - c \left(W + D(p) t_r \right) \\
& - D(p) \left(\frac{T - t_w}{t_r} \right) - D(p) T \\
& + \frac{1}{2} \left(\frac{p I_e D(p) M^2}{M} \right) / 2 + p I_e D(p) \left(\frac{T - t_w}{t_r} \right) M \\
& - c I_p \left(\frac{W + D(p) t_r}{t_d - M} \right) - \frac{D(p)}{\alpha^2} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& + \frac{D(p)}{\alpha^2} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) + \frac{1}{T} \left[D(p) p - s D(p) \left(\frac{T - t_w}{t_r} \right) + 2c D(p) + p I_e D(p) M \right] = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.2}}{\partial p} = \frac{1}{T} \left[\left(1 - e \right) D(p) T - \frac{F Y_2 t_r^2}{t_r \left(\frac{t_d - t_r}{t_r} \right) - \frac{1}{2} \left(\frac{t_d^2 - t_r^2}{t_r} \right)} \right] - H \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& - H \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) - H \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& - H \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) Y_3 - c Y_2 \left(\frac{t_r - \left(\frac{T - t_w}{t_r} \right)}{t_r} \right) - c D(p) Y_3 \\
& + p I_e Y_2 M \left(\frac{M}{2} + \left(\frac{T - t_w}{t_r} \right) \right) - p I_e D(p) M Y_3 - c I_p \left(\frac{W + Y_2 t_r}{t_d - M} \right) - \frac{Y_2}{\alpha^2} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& + \frac{Y_2}{\alpha^2} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) + \frac{Y_2}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) \\
& + \frac{D(p)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(\frac{t_w - t_d}{t_r} \right) - 1} \right) - \left(\frac{t_w - t_d}{t_r} \right) \right) Y_3 = 0
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.3}}{\partial t_r} = \frac{D \left(p \right) \left\{ T \right\} \left[\left\{ F t_r + H \left\{ \left\{ \left(t_d - t_r \right) \right\} \right\} + \frac{1}{\alpha} \left\{ 1 - e^{\alpha \left(t_w - t_d \right)} \right\} \right\} \right] Y_1}{\left(T - t_w \right) Y_1} + c \left(e^{\beta \left(t_r - t_d \right)} + Y_1 \right) + c \left(1 + Y_1 \right) \left[\frac{c I_p}{\alpha} \left(1 - e^{\alpha \left(t_w - t_d \right)} \right) Y_1 \right] - \frac{HW}{T} = 0 \\
& \frac{\partial \text{TP}_{2.3}}{\partial T} = \frac{1}{T^2} \left[D \left(p \right) p T - A - \frac{FD \left(p \right) t_r^2}{2} \right] - H \left\{ W t_r + D \left(p \right) t_r \left(t_d - t_r \right) \right\} + \frac{D \left(p \right)^2 \left(t_d^2 - t_r^2 \right)}{2} \\
& \left\{ + \frac{D \left(p \right)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right) \right\} - s D \left(p \right) \left(T - t_w \right)^2 / 2 - c \left\{ W + D \left(p \right) t_r - D \left(p \right) \left(T - t_w \right) \right\} \\
& + \frac{1}{2} \left\{ \left(p I_e D \left(p \right) M^2 \right) / 2 + p I_e D \left(p \right) \left(T - t_w \right) M \right\} - c I_p \frac{D \left(p \right)}{\alpha} \left\{ \frac{1}{\alpha} \left(e^{\alpha \left(t_w - M \right)} - 1 \right) - \left(t_w - M \right) \right\} \\
& \left. \right\} + \frac{1}{T} \left\{ D p - s D \left(T - t_w \right) + 3 c D + p I_e D M \right\} = 0 \end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.3}}{\partial p} = \frac{1}{T} \left[\left(1 - e \right) D \left(p \right) T - \frac{F Y_2 t_r^2}{2} \right] - H Y_2 \left\{ t_r \left(t_d - t_r \right) - \frac{1}{2} \left(t_d^2 - t_r^2 \right) \right\} \\
& + \frac{1}{\alpha} \left\{ \frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right\} \text{nonumber} \\
& + \frac{1}{\alpha} \left\{ \frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right\} \text{nonumber} \\
& + H \frac{D \left(p \right)}{\alpha} \left\{ e^{\alpha \left(t_w - t_d \right)} - 1 \right\} Y_3 - s Y_2 \left(T - t_w \right)^2 / 2 + s D \left(p \right) \left(T - t_w \right) Y_3 \\
& \text{nonumber} \\
& + c Y_2 \left\{ t_r - \left(T - t_w \right) \right\} \text{nonumber} \\
& + c D \left(p \right) Y_3 + p I_e Y_2 M \left\{ M / 2 + \left(T - t_w \right) \right\} - p I_e D \left(p \right) M Y_3 \\
& \text{nonumber} \\
& \left. \right\} - c I_p \frac{Y_2}{\alpha} \left\{ \frac{1}{\alpha} \left(e^{\alpha \left(t_w - M \right)} - 1 \right) - \left(t_w - M \right) \right\} - c I_p \frac{D \left(p \right)}{\alpha} \left(e^{\alpha \left(t_w - M \right)} - 1 \right) Y_3 \right] = 0 \text{nonumber}
\end{aligned}$$

(42)

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.4}}{\partial t_r} = \frac{-D \left(p \right) T \left[F t_r + H \left(t_d - t_r \right) + \frac{1}{\alpha} \left(1 - e^{\alpha \left(t_w - t_d \right)} \right) Y_1 \right]}{e^{\beta \left(t_r - t_d \right)} + Y_1} + c \left(1 + Y_1 \right) \frac{\partial \text{TP}_{2.4}}{\partial T} \\
& = \frac{HW}{T} = 0 \quad \frac{\partial \text{TP}_{2.4}}{\partial T} = - \frac{1}{T^2} \left[D \left(p \right) p T - A \frac{FD \left(p \right) t_r^2}{2} \right] \\
& - H \left(W t_r + D \left(p \right) t_r \left(t_d - t_r \right) \right) - \frac{D \left(p \right)}{2} \left(t_d^2 - t_r^2 \right) \\
& + \frac{D \left(p \right)}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right) - s D \left(p \right) \left(T - t_w \right)^2 / 2 \\
& - \left(W + D \left(p \right) t_r - D \left(p \right) \left(T - t_w \right) \right) \left(\left(p I_e D \left(p \right) t_w^2 \right) / 2 + p I_e D \left(p \right) t_w \left(M - t_w \right) + p I_e D \left(p \right) \left(T - t_w \right) M \right) \\
& + \left(D p - s D \left(T - t_w \right) + 2 c D + p I_e D M \right) = 0 \quad \text{end{aligned}}
\end{aligned}$$

$$\begin{aligned}
& \frac{\partial \text{TP}_{2.4}}{\partial p} = \frac{1}{T} \left[\left(1 - e \right) D \left(p \right) T - \frac{F Y_2 t_r^2}{2} \right] \\
& - \frac{H Y_2}{2} \left(t_r \left(t_d - t_r \right) - \frac{1}{2} \left(t_d^2 - t_r^2 \right) \right) + \frac{1}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) - \left(t_w - t_d \right) \right) \\
& - \frac{H}{\alpha} \left(\frac{1}{\alpha} \left(e^{\alpha \left(t_w - t_d \right)} - 1 \right) Y_3 - s Y_2 \left(T - t_w \right)^2 / 2 + s D \left(p \right) \left(T - t_w \right) Y_3 \right) \\
& - 2 c D \left(p \right) Y_3 - \frac{2 c Y_2}{2} \left(t_r - \left(T - t_w \right) \right) - 2 c D \left(p \right) Y_3 - \frac{2 c Y_2}{2} T + p I_e Y_2 \left(\left(p I_e D \left(p \right) t_w^2 \right) / 2 + t_w \left(M - t_w \right) + \left(T - t_w \right) M \right) \\
& - p I_e D \left(p \right) t_w Y_3 \right] = 0 \quad \text{end{aligned}}
\end{aligned}$$

(43)

where

$$\begin{aligned}
X_1 &= \frac{D(p)}{D(p) + \alpha W e^{\alpha(t_d - t_r)}}, \quad X_2 = Y_2 = \frac{\partial D(p)}{\partial p} = -ekp^{-\left(e+1\right)}, \quad X_3 = \frac{-D(p) W e^{\alpha(t_d - t_r)} \{e X_2 \left(D(p) + \alpha W e^{\alpha(t_d - t_r)} \right)\}}{D(p) + \alpha W e^{\alpha(t_d - t_r)} \left(W + D(p) e^{\alpha(t_r - t_d)} \right)}, \quad \text{and} \\
Y_3 &= \frac{-W Y_2}{D(p) \left(D(p) + \alpha W e^{\alpha(t_d - t_r)} \left(W + D(p) e^{\alpha(t_r - t_d)} \right) \right)}
\end{aligned}$$

Rights and permissions

[Reprints and permissions](#)

About this article

Cite this article

Jaggi, C.K., Tiwari, S. & Goel, S.K. Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. *Ann Oper Res* **248**, 253–280 (2017). <https://doi.org/10.1007/s10479-016-2179-3>

Published

05 April 2016

Issue Date

January 2017

DOI

<https://doi.org/10.1007/s10479-016-2179-3>

Keywords

[Inventory](#)

[Non-instantaneous deterioration](#)

[Two-warehouse](#)

[Permissible delay](#)

[Shortages](#)

Search

Search by keyword or author



Navigation

[Find a journal](#)

[Publish with us](#)

[Track your research](#)

