Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities

Original Paper | Published: 05 April 2016

Volume 248, pages 253–280, (2017) Cite this article

Annals of Operations Research

Aims and scope →

Submit manuscript →

Chandra K. Jaggi orcid.org/0000-0001-6179-8376[1], Sunil Tiwari[1] & Satish K. Goel[1]

Abstract

The formulation of classical deteriorating inventory models is done with the common unrealistic assumption that all the items start deteriorating as soon as they arrive in the warehouse. On the contrary, in a realistic environment, it has been observed that there are several items that do not deteriorate immediately. Items such as dry fruits, potatoes, yams and even some fruits and vegetables have a shelf life and deteriorate only after a time lag. Apart from this, in today's competitive business world, the supplier usually offers a trade credit period to his retailers to attract more sales and the retailer considers it as an alternative to price discount. The present research proposes a two warehouse inventory model for non-instantaneous deteriorating items under trade credit based on the above phenomena, where the demand rate is assumed to be a function of the selling price. Further, shortages are completely backlogged and the interest on shortages at the beginning of the cycle has also been considered. The objective of the study is to determine the retailer's optimal replenishment policies that maximize the average profit per unit time. Conclusively, a numerical example is presented to illustrate the applicability of the proposed model. Sensitivity analysis on key parameters is provided to reveal the managerial insights.

References

Aggarwal, S. P., & Jaggi, C. K. (1995). Ordering policies of deteriorating items under permissible delay in payments. *Journal of the Operational Research Society*, *46*(5), 658–662.

Article Google Scholar

Arcelus, F. J., & Srinivasan, G. (1993). Delay of payments for extraordinary purchases. *Journal of the Operational Research Society*, *44*(8), 785–795.

Article Google Scholar

Bakker, M., Riezebos, J., & Teunter, R. H. (2012). Review of inventory systems with deterioration since 2001. *European Journal of Operational Research*, *221*(2), 275–284.

Article Google Scholar

Chang, C. T., Teng, J. T., & Goyal, S. K. (2010). Optimal replenishment policies for non-instantaneous deteriorating items with stock-dependent demand. *International Journal of Production Economic*, *123*(1), 62–68.

Article Google Scholar

Chen, S.-C., Cárdenas-Barrón, L. E., & Teng, J. T. (2014). Retailers economic order quantity when the supplier offers conditionally permissible delay in payments link to order quantity. *International Journal of Production Economics*, *155*, 284–291.

Article Google Scholar

Chung, K. J. (1998). A theorem on the determination of economic order quantity under conditions of permissible delay in payments. *Computer and Operations Research*, *25*(1), 49–52.

Article Google Scholar

Chung, K. J., & Liao, J. J. (2004). Lot-sizing decisions under trade credit depending on the ordering quantity. *Computers and Operations Research*, *31*(6), 909–928.

Article Google Scholar

Chung, K. J., & Liao, J. J. (2006). The optimal ordering policy in a DCF analysis for deteriorating items under trade credit depending on the ordering quantity. *International Journal of Production Economics*, *100*(1), 116–130.

Article Google Scholar

Chung, K. J., & Liao, J. J. (2009). The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. *European Journal of Operational Research*, *196*(2), 563–568.

Article Google Scholar

Chung, K. J., & Liao, J. J. (2011). The simplified solution algorithm for an integrated supplier–buyer inventory model with two-part trade credit in a supply chain system. *European Journal of Operational Research*, *213*(1), 156–165.

Article Google Scholar

Chung, K. J., & Cárdenas-Barrón, L. E. (2013). The simplified solution procedure for deteriorating itemsunder stock-dependent demand and two-level trade-credit in the supply chain management. *Applied Mathematical Modelling*, *37*(7), 4653–4660.

Article Google Scholar

Chung, K. J., Cárdenas-Barrón, L. E., & Ting, P. S. (2014). An inventory model with non-instantaneous receipt and exponentially deteriorating items for an integrated three layer supply chain system under two levels of trade-credit. *International Journal of Production Economics*, *155*, 310–317.

Article Google Scholar

Covert, R. P., & Philip, G. C. (1973). An EOQ model for items with Weibull distribution deterioration. *American Institute of Industrial Engineering Transactions*, *5*(4), 323–326.

Google Scholar

Dye, C. Y. (2013). The effect of preservation technology investment on a non-instantaneous deteriorating inventory model. *Omega*, *41*(5), 872–880.

Article Google Scholar

Geetha, K. V., & Uthayakumar, R. (2010). Economic design of an inventory policy for non-instantaneous deteriorating items under permissible delay in payments. *Journal of Computational and Applied Mathematics*, *233*(10), 2492–2505.

Article Google Scholar

Ghare, P. M., & Shrader, G. F. (1963). A model for exponentially decaying inventories. *Journal of Industrial Engineering*, *14*(5), 238–243.

Google Scholar

Goyal, S. K. (1985). Economic order quantity under conditions of permissible delay in payments. *Journal of the Operational Research Society*, *36*(4), 335–338.

Article Google Scholar

Goyal, S. K., & Giri, B. C. (2001). Recent trends in modelling of deteriorating inventory. *European Journal of Operational Research*, *134*(1), 1–16.

Article Google Scholar

Haley, C. W., & Higgins, R. C. (1973). Inventory policy and trade credit financing. *Management Science*, *20*(4), 464–471.

Article Google Scholar

Hartely, V. R. (1976). *Operations research: A managerial emphasis*. California: Good Year Publishing Company.

Google Scholar

Hsieh, T. P., Dye, C. Y., & Ouyang, L. Y. (2008). Determining optimal lot size for a two-warehouse system with deterioration and shortages using net present value. *European Journal of Operational Research*, *191*(1), 182–192.

Article Google Scholar

Hwang, H., & Shinn, S. W. (1997). Retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. *Computers and Operations Research*, *24*(6), 539–547.

Article Google Scholar

Huang, K. N., & Liao, J. J. (2008). A simple method to locate the optimal solution for exponentially deteriorating items under trade credit financing. *Computers and Mathematics with Applications*, *56*(4), 965–977.

Article Google Scholar

Jaggi, C. K., & Aggarwal, S. P. (1994). Credit financing in economic ordering policies of deteriorating items. *International Journal of Production Economics*, *34*(2), 151–155.

Article Google Scholar

Jaggi, C. K., & Verma, P. (2010). An optimal replenishment policy for non-instantaneous deteriorating items with two storage facilities. *International Journal of Services Operations and Informatics*, *5*(3), 209–230.

Article Google Scholar

Jaggi, C. K., & Tiwari, S. (2014). Two-Warehouse inventory model for non-instantaneous deteriorating items with price dependent demand and time-varying holding cost. In Om Parkash (Ed.), *Mathematical modeling and applications* (pp. 225–238). Germany: LAMBERT Academic Publishers.

Google Scholar

Jaggi, C. K., Sharma, A., & Tiwari, S. (2015a). Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand under permissible delay in payments: A new approach. *International Journal of Industrial Engineering Computations*, *6*(4), 481–502.

Article Google Scholar

Jaggi, C. K., Tiwari, S., & Shafi, A. (2015b). Effect of deterioration on two-warehouse inventory model with imperfect quality. *Computers and Industrial Engineering*, *88*, 378–385.

Article Google Scholar

Jamal, A. M. M., Sarker, B. R., & Wang, S. (1997). An ordering policy for deteriorating items with allowable shortage and permissible delay in payment. *Journal of the Operational Research Society*, *48*(8), 826–833.

Article Google Scholar

Jamal, A. M. M., Sarker, B. R., & Wang, S. (2000). Optimal payment time for a retailer under permitted delay of payment by the wholesaler. *International Journal of Production Economics*, *66*(1), 59–66.

Article Google Scholar

Lee, C. C. (2006). Two-warehouse inventory model with deterioration under FIFO dispatching policy. *European Journal of Operational Research*, *174*(2), 861–873.

Article Google Scholar

Lee, C. C., & Hsu, S. L. (2009). A two-warehouse production model for deteriorating inventory items with time-dependent demands. *European Journal of Operational Research*, *194*(3), 700–710.

Article Google Scholar

Li, Y., Zhen, X., & Cai, X. (2014). Trade credit insurance, capital constraint, and the behavior of manufacturers and banks. *Annals of Operations Research*, 1–20. doi:10.1007/s10479-014-1602-x.

Liang, Y., & Zhou, F. (2011). A two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. *Applied Mathematical Modelling*, *35*(5), 2221–2231.

Article Google Scholar

Liao, J. J., & Chung, K. J. (2009). An EOQ model for deterioration items under trade credit policy in a supply chain system. *Journal of the Operations Research Society of Japan*, *52*(1), 46.

Google Scholar

Liao, J. J., Huang, K. N., & Chung, K. J. (2012). Lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit. *International Journal of Production Economics*, *137*(1), 102–115.

Article Google Scholar

Liao, J. J., Huang, K. N., & Chung, K. J. (2013). Optimal pricing and ordering policy for perishable items with limited storage capacity and partial trade credit. *IMA Journal of Management Mathematics*, *24*(1), 45–61.

Article Google Scholar

Mahata, G. C. (2012). An EPQ-based inventory model for exponentially deteriorating items under retailer partial trade credit policy in supply chain. *Expert Systems with Applications*, *39*(3), 3537–3550.

Article Google Scholar

Maihami, R., & Kamalabadi, I. N. (2012a). Joint pricing and inventory control for non-instantaneous deteriorating items with partial backlogging and time and price dependent demands. *International Journal of Production Economics*, *136*(1), 116–122.

Article Google Scholar

Maihami, R., & Kamalabadi, I. N. (2012b). Joint control of inventory and its pricing for non-instantaneously deteriorating items under permissible delay in payments and partial backlogging. *Mathematical and Computer Modelling*, *55*(5–6), 1722–1733.

Article Google Scholar

Niu, B., & Xie, J. (2008). A note on Two-warehouse inventory model with deterioration under FIFO dispatches policy. *European Journal of Operational Research*, *190*(2), 571–577.

Article Google Scholar

Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2006). A study on an inventory model for non-instantaneous deteriorating items with permissible delay in payments. *Computers and Industrial Engineering*, *51*(4), 637–651.

Article Google Scholar

Ouyang, L. Y., Wu, K. S., & Yang, C. T. (2008). Retailer's ordering policy for non-instantaneous deteriorating items with quantity discount, stock dependent demand and stochastic backorder rate. *Journal of the Chinese Institute of Industrial Engineers*, *25*(1), 62–72.

Article Google Scholar

Ouyang, L. Y., Yang, C. T., Chan, Y. L., & Cárdenas-Barrón, L. E. (2013). A comprehensive extension of theoptimal replenishment decisions under two-level of trade-credit policy depending on the order quantity. *Applied Mathematics and Computation*, *224*, 268–277.

Article Google Scholar

Raafat, F. (1991). Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, *42*(1), 27–37.

Article Google Scholar

Sarker, B. R., Jamal, A. M. M., & Wang, S. (2001). Optimal payment time under permissible delay in payment for products with deterioration. *Production Planning and Control*, *11*(4), 380–390.

Article Google Scholar

Sarkar, B., Saren, S., & Cárdenas-Barrón, L. E. (2014). An inventory model with trade-credit policy and variable deterioration for fixed lifetime products. *Annals of Operations Research*, *229*(1), 677–702.

Article Google Scholar

Shah, N. H., Soni, H. N., & Patel, K. A. (2013). Optimizing inventory and marketing policy for non-instantaneous deteriorating items with generalized type deterioration and holding cost rates. *Omega*, *41*(2), 421–430.

Article Google Scholar

Soni, H., & Patel, K. (2012). Optimal pricing and inventory policies for non-instantaneous deteriorating items with permissible delay in payment: Fuzzy expected value model. *International Journal of Industrial Engineering Computations*, *3*(3), 281–300.

Article Google Scholar

Thangam, A. (2012). Optimal price discounting and lot-sizing policies for perishable items in a supply chain under advance payment scheme and two-echelon trade credits. *International Journal of Production Economics*, *139*(2), 459–472.

Article Google Scholar

Wu, K. S., Ouyang, L. Y., & Yang, C. T. (2006). An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. *International Journal of Production Economics*, *101*(2), 369–384.

Article Google Scholar

Wu, K. S., Ouyang, L. Y., & Yang, C. T. (2009). Coordinating replenishment and pricing policies for non-instantaneous deteriorating items with price-sensitive demand. *International Journal of System Sciences*, *40*(12), 1273–1281.

Article Google Scholar

Wu, J., Ouyang, L. Y., Cárdenas-Barrón, L. E., & Goyal, S. K. (2014). Optimal credit period and lot size for deteriorating items with expiration dates under two-level trade-credit financing. *European Journal of Operational Research*, *237*(3), 898–908.

Article Google Scholar

Yang, H. L. (2004). Two-warehouse inventory models for deteriorating items with shortages under inflation. *European Journal of Operational Research*, *157*(1), 344–356.

Article Google Scholar

Yang, H. L. (2006). Two-warehouse partial backlogging inventory models for deteriorating items under inflation. *International Journal of Production*

Economics, *103*(1), 362–370.

Article Google Scholar

Yang, H. L. (2012). Two-warehouse partial backlogging inventory models with three parameters Weibull distribution deterioration under inflation. *International Journal of Production Economics*, *138*(1), 107–116.

Article Google Scholar

Zhou, Y. W., & Yang, S. L. (2005). A two-warehouse inventory model for items with stock-level dependent demand rate. *International Journal of Production Economics*, *95*(2), 215–228.

Article Google Scholar

Zhong, Y. G., & Zhou, Y. W. (2013). Improving the supply chain's performance through trade credit under inventory-dependent demand and limited storage capacity. *International Journal of Production Economics*, *143*(2), 364–370.

Article Google Scholar

Acknowledgments

The authors would like to thank the Editor Endre Boros and anonymous referee for their valuable and constructive comments on earlier versions of our paper, which have led to a significant improvement in the manuscript. The first author acknowledges the support of the University Grants Commission through University of Delhi (Research Grant No. RC/2015/9677). The second author would like to thank University Grant Commission (UGC) for providing the Non-NET fellowship to accomplish this research.

Author information

Authors and Affiliations

Department of Operational Research, Faculty of Mathematical Sciences, New Academic Block, University of Delhi, Delhi, 110007, India

Chandra K. Jaggi, Sunil Tiwari & Satish K. Goel

Corresponding author

Correspondence to Chandra K. Jaggi.

Appendices

Appendix 1

For Case 1, the necessary conditions for maximizing the total average profit is given by

$$\begin{aligned} \frac{\partial \textit{TP}_{1.1} }{\partial t_r }= & {} \frac{1}{T}\left[\frac{D\left(p \right) }{\beta }\left({cI_p +F} \right) \left({1-e^{\beta \left({t_r -t_d } \right) }} \right) \right. \nonumber \\&\left. +\,\left({cI_p -H} \right) \left\{ {We^{-\alpha \left({t_r -t_d } \right) }+\frac{D\left(p \right) }{\alpha }\left\{ {\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) \left({1-X_1 } \right) } \right\} } \right\} \right. \nonumber \\&\left. -\,FD\left(p \right) t_d e^{\beta \left({t_r -t_d } \right) }+sD\left(p \right) \left({T-t_w } \right) X_1 -cD\left(p \right) \left({e^{\beta \left({t_r -t_d } \right) }+X_1 } \right) \right. \nonumber \\&\left. -\,pI_e D\left(p \right) MX_1 -cI_p D\left(p \right) e^{\beta \left({t_r -t_d } \right) }\left({t_d -M} \right) \right] =0 \nonumber \\ \frac{\partial \textit{TP}_{1.1} }{\partial T}= & {} -\frac{1}{T^{2}}\left[D\left(p \right) pT\right. \nonumber \\&\left. -\,A-F\left\{ {Zt_d -\frac{D\left(p \right) t_d^2 }{2}-Wt_d -\frac{D\left(p \right) }{\beta }\left({t_r -t_d +\frac{1}{\beta }-\frac{e^{\beta \left({t_r -t_d } \right) }}{\beta }} \right) } \right\} \right. \nonumber \\&-\,H\left\{ {Wt_d +\frac{W}{\alpha }\left({1-e^{\alpha \left({t_d -t_r } \right) }} \right) +\frac{D\left(p \right) }{\alpha }\left({t_r -t_w -\frac{1}{\alpha }+\frac{e^{\alpha \left({t_w -t_r } \right) }}{\alpha }} \right) } \right\} \nonumber \\&-\,sD\left(p \right) \left({T-t_w } \right) ^{2}/2 \nonumber \\&-\,c\left\{ {W+D\left(p \right) t_d +\frac{D\left(p \right) }{\beta }\left({e^{\beta

\left({t_r -t_d } \right) }-1} \right) -D\left(p \right) \left({T-t_w } \right) } \right\} \nonumber \\&+\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M} \right\} \nonumber \\&-\,cI_p \left\{ Z\left({t_d -M} \right) +\frac{W}{\alpha }\left({1-e^{\alpha \left({t_d -t_r } \right) }} \right) +\frac{D\left(p \right) }{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) \right. \nonumber \\&\left. -\,\frac{D\left(p \right) }{\alpha }\left({t_w -t_r } \right) -\frac{D\left(p \right) }{2}\left({t_d^2 -M^{2}} \right) \right. \nonumber \\&\left. {\left. { +\,\frac{D\left(p \right) }{\beta ^{2}}\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -\frac{D\left(p \right) }{\beta }\left({t_r -t_d } \right) } \right\} } \right] +\frac{1}{T}\left\{ D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) \right. \nonumber \\&\left. +\,2cD\left(p \right) +pI_e D\left(p \right) M \right\} =0 \nonumber \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{1.1} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T+F\left\{ {\frac{t_d^2 }{2}+\frac{1}{\beta }\left({t_r -t_d +\frac{1}{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) } \right) } \right\} } \right. X_2 \nonumber \\&-\,\frac{H}{\alpha }\left\{ {t_r -t_w -\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) } \right\} X_2 \nonumber \\&-\,\frac{HD\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 +s\left({T-t_w } \right) \left\{ {D\left(p \right) X_3 -\frac{\left({T-t_w } \right) }{2}X_2 } \right\} \nonumber \\&-\,c\left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -\left({T-t_w } \right) } \right\} X_2 \nonumber \\&-\,cD\left(p \right) X_3 +\left\{ {\frac{pI_e M^{2}}{2}+pI_e \left({T-t_w } \right) M} \right\} X_2 -pI_e D\left(p \right) MX_3 +\frac{D\left(p \right) I_e M^{2}}{2}\nonumber \\&+\,D\left(p \right) I_e \left({T-t_w } \right) M \nonumber \\&-\,cI_p \left\{ {\frac{1}{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) } \right. \left. -\frac{1}{\alpha }\left({t_w -t_r } \right) -\frac{1}{2}\left({t_d^2 -M^{2}} \right) +\frac{1}{\beta ^{2}}\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) \right. \nonumber \\&\left. -\,\frac{1}{\beta }\left({t_r -t_d } \right) \right\} X_2 -\frac{cI_p D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 \nonumber \\&\left. { -\,\left\{ {cI_p \left({t_d -M} \right) +Ft_d } \right\} \left({t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) } \right) X_2 } \right] =0 \end{aligned}$$

(36)

$$\begin{aligned} \frac{\partial \textit{TP}_{1.2} }{\partial t_r }= & {} \frac{1}{T}\left[\frac{FD\left(p \right) }{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) \right. \\&\left. -\,\left({cI_p +H} \right) \left\{ {We^{-\alpha \left({t_r -t_d } \right) }+\frac{D\left(p \right) }{\alpha }\left\{ {\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) \left({1-X_1 } \right) } \right\} } \right\} \right. \\&\left. -\,FD\left(p \right) t_d e^{\beta \left({t_r -t_d } \right) }+sD\left(p \right) \left({T-t_w } \right) X_1 -cD\left(p \right) \left({e^{\beta \left({t_r -t_d } \right) }+X_1 } \right) \right. \\&\left. -\,pI_e D\left(p \right) MX_1 -\frac{cI_p D\left(p \right) }{\beta }\left({e^{\beta \left({t_r -M} \right) }-1} \right) \right] =0 \\ \frac{\partial \textit{TP}_{1.2} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-F\left\{ {Zt_d -\frac{D\left(p \right) t_d^2 }{2}-Wt_d -\frac{D\left(p \right) }{\beta }\left({t_r -t_d +\frac{1}{\beta }-\frac{e^{\beta \left({t_r -t_d } \right) }}{\beta }} \right) } \right\} } \right. \\&-\,H\left\{ {Wt_d +\frac{W}{\alpha }\left({1-e^{\alpha \left({t_d -t_r } \right) }} \right) +\frac{D\left(p \right) }{\alpha }\left({t_r -t_w -\frac{1}{\alpha }+\frac{e^{\alpha \left({t_w -t_r } \right) }}{\alpha }} \right) } \right\} -sD\left(p \right) \left({T-t_w } \right) ^{2}/2 \\&-\,c\left\{ {W+D\left(p \right) t_d +\frac{D\left(p \right) }{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -D\left(p \right) \left({T-t_w } \right) } \right\} \\&+\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M} \right\} \\&\left. -\,cI_p \left\{ \frac{W}{\alpha }\left({e^{\alpha \left({t_d -M} \right) }-e^{\alpha \left({t_d -t_r } \right) }} \right) \right. \right. \\&\left. \left. +\frac{D\left(p \right) }{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) -\frac{D\left(p \right) }{\alpha }\left({t_w -t_r } \right) +\frac{D\left(p \right) }{\beta }\left\{ {\frac{1}{\beta }\left({e^{\beta \left({t_r -M} \right) }-1} \right) -\left({t_r -M} \right) } \right\} \right\} \right] \\&+\,\frac{1}{T}\left\{ {D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) +2cD\left(p \right) +pI_e D\left(p \right) M} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{1.2} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T+F\left\{ {\frac{t_d^2 }{2}+\frac{1}{\beta }\left({t_r -t_d +\frac{1}{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) } \right) } \right\} } \right. X_2 \nonumber \\&-\,\frac{H}

{\alpha }\left\{ {t_r -t_w -\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) } \right\} X_2 \nonumber \\&-\,\frac{HD\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 +s\left({T-t_w } \right) \left\{ {D\left(p \right) X_3 -\frac{\left({T-t_w } \right) }{2}X_2 } \right\} \nonumber \\&-\,c\left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -\left({T-t_w } \right) } \right\} X_2 \nonumber \\&-\,cD\left(p \right) X_3 -Ft_d \left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) } \right\} X_2 +\left\{ {\frac{pI_e M^{2}}{2}+pI_e \left({T-t_w } \right) M} \right\} X_2\nonumber \\&-\,pI_e D\left(p \right) MX_3 +\frac{D\left(p \right) I_e M^{2}}{2} \nonumber \\&+\,D\left(p \right) I_e \left({T-t_w } \right) M-cI_p \left\{ {\frac{1}{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) -\frac{1}{\alpha }\left({t_w -t_r } \right) } \right. \left. \left. +\frac{1}{\beta ^{2}}\left({e^{\beta \left({t_r -M} \right) }-1} \right) \right. \right. \nonumber \\&\left. \left. -\frac{1}{\beta }\left({t_r -M} \right) \right\} X_2 -\frac{cI_p D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 \right] =0 \end{aligned}$$

(37)

$$\begin{aligned} \frac{\partial \textit{TP}_{1.3} }{\partial t_r }= & {} \frac{1}{T}\left[{\frac{FD\left(p \right) }{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) -H\left\{ {We^{-\alpha \left({t_r -t_d } \right) }+\frac{D\left(p \right) }{\alpha }\left\{ {\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) \left({1-X_1 } \right) } \right\} } \right\} } \right. \\&\left. -\,FDt_d e^{\beta \left({t_r -t_d } \right) }+sD\left(p \right) \left({T-t_w } \right) X_1 -cD\left(p \right) \left({e^{\beta \left({t_r -t_d } \right) }+X_1 }\right) \right. \\&\left. -\,pI_e D\left(p \right) MX_1 -\frac{cI_p D\left(p \right) }{\alpha }\left\{ {\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) \left({X_1 -1} \right) } \right\} \right] =0 \\ \frac{\partial \textit{TP}_{1.3} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-F\left\{ {Zt_d -\frac{D\left(p \right) t_d^2 }{2}-Wt_d -\frac{D\left(p \right) }{\beta }\left({t_r -t_d +\frac{1}{\beta }-\frac{e^{\beta \left({t_r -t_d } \right) }}{\beta }} \right) } \right\} } \right. \\&-\,H\left\{ {Wt_d +\frac{W}{\alpha }\left({1-e^{\alpha \left({t_d -t_r } \right) }} \right) +\frac{D\left(p \right) }{\alpha }\left({t_r -t_w -\frac{1}{\alpha }+\frac{e^{\alpha \left({t_w -t_r } \right) }}{\alpha }} \right) } \right\} -sD\left(

p \right) \left({T-t_w } \right) ^{2}/2 \\&-\,c\left\{ {W+D\left(p \right) t_d +\frac{D\left(p \right) }{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -D\left(p \right) \left({T-t_w } \right) } \right\} \\&+\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M} \right\} \\&\left. {-\,cI_p \frac{D\left(p \right) }{\alpha }\left\{ {\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) -\left({t_w -t_r } \right) } \right\} } \right] +\frac{1}{T}\left\{ D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) \right. \\&\left. +\,2cD\left(p \right) +pI_e D\left(p \right) M \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{1.3} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T+F\left\{ {\frac{t_d^2 }{2}+\frac{1}{\beta }\left({t_r -t_d +\frac{1}{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) } \right) } \right\} } \right. X_2\nonumber \\&-\,\frac{H}{\alpha }\left\{ {t_r -t_w -\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) } \right\} X_2 \nonumber \\&-\,\frac{HD\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 +s\left({T-t_w } \right) \left\{ {D\left(p \right) X_3 -\frac{\left({T-t_w } \right) }{2}X_2 } \right\} \nonumber \\&-\,2c\left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -\left({T-t_w } \right) } \right\} X_2\nonumber \\&-\,cD\left(p \right) X_3 -Ft_d \left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) } \right\} X_2 +\left\{ {\frac{pI_e M^{2}}{2}+pI_e \left({T-t_w } \right) M} \right\} X_2 \nonumber \\&-\,pI_e D\left(p \right) MX_3 +\frac{D\left(p \right) I_e M^{2}}{2} \nonumber \\&\left. +\,D\left(p \right) I_e \left({T-t_w } \right) M-cI_p \left\{ \frac{1}{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) -\frac{1}{\alpha }\left({t_w -t_r } \right) \right\} X_2 \right. \nonumber \\&\left. -\,\frac{cI_p D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 \right] =0 \end{aligned}$$

(38)

$$\begin{aligned} \frac{\partial \textit{TP}_{1.4} }{\partial t_r }= & {} \frac{1}{T}\left[{\frac{FD\left(p \right) }{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) -H\left\{ {We^{-\alpha \left({t_r -t_d } \right) }+\frac{D\left(p \right) }{\alpha }\left\{ {\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) \left(

{1-X_1 } \right) } \right\} } \right\} } \right. \\&\left. { -\,FD\left(p \right) t_d e^{\beta \left({t_r -t_d } \right) }+sD\left(p \right) \left({T-t_w } \right) X_1 -cD\left(p \right) \left({e^{\beta \left({t_r -t_d } \right) }+X_1 } \right) +pI_e D\left(p \right) \left({3t_w -2M} \right) X_1 } \right] =0 \\ \frac{\partial \textit{TP}_{1.4} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-F\left\{ {Zt_d -\frac{D\left(p \right) t_d^2 }{2}-Wt_d -\frac{D\left(p \right) }{\beta }\left({t_r -t_d +\frac{1}{\beta }-\frac{e^{\beta \left({t_r -t_d } \right) }}{\beta }} \right) } \right\} } \right. \\&-\,H\left\{ {Wt_d +\frac{W}{\alpha }\left({1-e^{\alpha \left({t_d -t_r } \right) }} \right) +\frac{D\left(p \right) }{\alpha }\left({t_r -t_w -\frac{1}{\alpha }+\frac{e^{\alpha \left({t_w -t_r } \right) }}{\alpha }} \right) } \right\} -sD\left(p \right) \left({T-t_w } \right) ^{2}/2 \\&-\,c\left\{ {W+D\left(p \right) t_d +\frac{D\left(p \right) }{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -D\left(p \right) \left({T-t_w } \right) } \right\} \\&\left. { +\,\left\{ {\left({pI_e D\left(p \right) t_w^2 } \right) /2-pI_e D\left(p \right) t_w \left({M-t_w } \right) +pI_e D\left(p \right) \left({T-t_w } \right) M} \right\} } \right] \\&+\,\frac{1}{T}\left\{ {D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) +2cD\left(p \right) +pI_e D\left(p \right) M} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{1.4} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T+F\left\{ {\frac{t_d^2 }{2}+\frac{1}{\beta }\left({t_r -t_d +\frac{1}{\beta }\left({1-e^{\beta \left({t_r -t_d } \right) }} \right) } \right) } \right\} } \right. X_2 \nonumber \\&-\,\frac{H}{\alpha }\left\{ {t_r -t_w -\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_r } \right) }} \right) } \right\} X_2 \nonumber \\&-\,\frac{HD\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_r } \right) }-1} \right) X_3 +s\left({T-t_w } \right) \left\{ {D\left(p \right) X_3 -\frac{\left({T-t_w } \right) }{2}X_2 } \right\} \nonumber \\&-\,c\left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) -\left({T-t_w } \right) } \right\} X_2 \nonumber \\&-\,cD\left(p \right) X_3 +\left\{ {\frac{pI_e M^{2}}{2}+pI_e \left({T-t_w } \right) M} \right\} X_2 +\left\{ \left({I_e D\left(p \right) t_w^2 } \right) /2+pI_e D\left(p \right) t_w X_3 \right. \nonumber \\&\left. -\,I_e D\left(p \right) t_w \left({M-t_w } \right) \right. \nonumber \\&\left. \left. -pI_e D\left(p \right) \left({M-t_w } \right) X_3 -Ft_d \left\{ {t_d +\frac{1}{\beta }\left({e^{\beta \left({t_r -t_d } \right) }-1} \right) } \right\} X_2 +pI_e D\left(p \right) t_w X_3 \right. \right. \nonumber

\\&\left. \left. +\,I_e D\left(p \right) \left({T-t_w } \right) M-pI_e D\left(p \right) MX_3 \right\} \right] =0 \nonumber \\ \end{aligned}$$

(39)

Appendix 2

For Case 2, the necessary conditions for maximizing the total average profit is given by

$$\begin{aligned} \frac{\partial \textit{TP}_{2.1} }{\partial t_r }= & {} \frac{-D\left(p \right) }{T}\left[{Ft_r +H\left\{ {\left({t_d -t_r } \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} } \right. -s\left({T-t_w } \right) Y_1 \\&-\,c\left({e^{\beta \left({t_r -t_d } \right) }+Y_1 } \right) +c\left({1+Y_1 } \right) \\&\left. { -\,pI_e MY_1 +cI_p \left\{ {\left({t_d -t_r } \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} +\left({t_r -M} \right) } \right] -\frac{HW}{T}=0 \\ \frac{\partial \textit{TP}_{2.1} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-\frac{FD\left(p \right) t_r^2 }{2}} \right. -H\left\{ Wt_r +D\left(p \right) t_r \left({t_d -t_r } \right) -\frac{D\left(p \right) }{2}\left({t_d^2 -t_r^2 } \right) \right. \\&\left. +\,\frac{D\left(p \right) }{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) \right\} \\&-\,sD\left(p \right) \left({T-t_w } \right) ^{2}/2-c\left({W+D\left(p \right) t_r -D\left(p \right) \left({T-t_w } \right) } \right) \\&+\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M } \right\} -cI_p \left\{ W\left({t_d -M} \right) +D\left(p \right) t_r \left({t_d -t_r } \right) \right. \\&\left. -\,\frac{D\left(p \right) }{2}\left({t_d -t_r } \right) ^{2} \right. \\&+\left. \left. \frac{D\left(p \right) }{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\frac{D\left(p \right) }{\alpha }\left({t_w -t_d } \right) +\frac{D\left(p \right) }{2}\left({t_r -M} \right) ^{2} \right\} \right] \\&+\,\frac{1}{T}\left\{ {D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) +2cD\left(p \right) +pI_e D\left(p \right) M} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{2.1} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T-\frac{FY_2 t_r^2 }{2}} \right. -

HY_2 \left\{ t_r \left({t_d -t_r } \right) -\frac{1}{2}\left({t_d^2 -t_r^2 } \right) \right. \nonumber \\&\left. +\,\frac{1}{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) \right\} -H\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 \nonumber \\&-\,sY_2 \left({T-t_w } \right) ^{2}/2+sD\left(p \right) \left({T-t_w } \right) Y_3 -cY_2 \left({t_r -\left({T-t_w } \right) } \right) -cD\left(p \right) Y_3 -pI_e D\left(p \right) MY_3 \nonumber \\&+\,Y_2 \left\{ {\left({pI_e M^{2}} \right) /2+pI_e \left({T-t_w } \right) M } \right\} -cI_p \left\{ {Y_2 t_r \left({t_d -t_r } \right) -\frac{Y_2 }{2}\left({t_d -t_r } \right) ^{2}} \right. +\frac{Y_2 }{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) \nonumber \\&+\,\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 \nonumber \\&\qquad \left. {\left. {-\,\frac{Y_2 }{\alpha }\left({t_w -t_d } \right) +\frac{Y_2 }{2}\left({t_r -M} \right) ^{2}} \right\} } \right] =0 \end{aligned}$$

(40)

$$\begin{aligned} \frac{\partial \textit{TP}_{2.2} }{\partial t_r }= & {} \frac{-D\left(p \right) }{T}\left[{Ft_r +H\left\{ {\left({t_d -t_r } \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} } \right. -s\left({T-t_w } \right) Y_1 \\&-\,c\left({e^{\beta \left({t_r -t_d } \right) }+Y_1 } \right) +c\left({1+Y_1 } \right) \\&\left. { -\,pI_e MY_1 +cI_p \left\{ {\left({t_d -M} \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} } \right] -\frac{HW}{T}=0 \\ \frac{\partial \textit{TP}_{2.2} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-\frac{FD\left(p \right) t_r^2 }{2}} \right. -H\left\{ Wt_r +D\left(p \right) t_r \left({t_d -t_r } \right) \right. \\&\left. -\,\frac{D\left(p \right) }{2}\left({t_d^2 -t_r^2 } \right) +\frac{D\left(p \right) }{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) \right\} \\&-\,sD\left(p \right) \left({T-t_w } \right) ^{2}/2-c\left(W+D\left(p \right) t_r \right. \\&\left. -\,D\left(p \right) \left({T-t_w } \right) -D\left(p \right) T \right) \\&+\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M } \right\} -cI_p \left\{ \left({W+D\left(p \right) t_r } \right) \left({t_d -M} \right) \right. \\&\left. -\,\frac{D\left(p \right) }{2}\left({t_d^2 -M^{2}} \right) +\frac{D\left(p \right) }{\alpha ^{2}}\left({e^{\alpha

\left({t_w -t_d } \right) }-1} \right) \right. \\&\left. {\left. {-\,\frac{D\left(p \right) }{\alpha }\left({t_w -t_d } \right) } \right\} } \right] +\frac{1}{T}\left\{ {D\left(p \right) p-sD\left(p \right) \left({T-t_w } \right) +2cD\left(p \right) +pI_e D\left(p \right) M} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{2.2} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T-\frac{FY_2 t_r^2 }{2}} \right. -HY_2 \left\{ t_r \left({t_d -t_r } \right) -\frac{1}{2}\left({t_d^2 -t_r^2 } \right) \right. \nonumber \\&\left. +\,\frac{1}{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) \right\} -H\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 \nonumber \\&-\,sY_2 \left({T-t_w } \right) ^{2}/2+sD\left(p \right) Y_3 -cY_2 \left\{ {t_r -\left({T-t_w } \right) } \right\} -cD\left(p \right) Y_3 \nonumber \\&+\,pI_e Y_2 M\left\{ {M/2+\left({T-t_w } \right) } \right\} -pI_e D\left(p \right) MY_3 \nonumber \\&\left. -\,cI_p \left\{ \left({W+Y_2 t_r } \right) \left({t_d -M} \right) -\frac{Y_2 }{2}\left({t_d^2 -M^{2}} \right) +\frac{Y_2 }{\alpha ^{2}}\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\frac{Y_2 }{\alpha }\left({t_w -t_d } \right) \right. \right. \nonumber \\&\left. \left. +\,\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 \right\} \right] =0 \end{aligned}$$

(41)

$$\begin{aligned} \frac{\partial \textit{TP}_{2.3} }{\partial t_r }= & {} \frac{-D\left(p \right) }{T}\left[{Ft_r +H\left\{ {\left({t_d -t_r } \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} } \right. -s\left({T-t_w } \right) Y_1\\&-\,c\left({e^{\beta \left({t_r -t_d } \right) }+Y_1 } \right) +c\left({1+Y_1 } \right) \\&\left. { -\,pI_e MY_1 +\frac{cI_p }{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right] -\frac{HW}{T}=0 \\ \frac{\partial \textit{TP}_{2.3} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-\frac{FD\left(p \right) t_r^2 }{2}} \right. -H\left\{ Wt_r +D\left(p \right) t_r \left({t_d -t_r } \right) \right. \\&\left. -\,\frac{D\left(p \right) }{2}\left({t_d^2 -t_r^2 } \right) \right. \\&\left. {+\,\frac{D\left(p \right) }{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) } \right\} -sD\left(

p \right) \left({T-t_w } \right) ^{2}/2\\&-\,c\left\{ {W+D\left(p \right) t_r -D\left(p \right) \left({T-t_w } \right) } \right\} \\&\left. +\,\left\{ {\left({pI_e D\left(p \right) M^{2}} \right) /2+pI_e D\left(p \right) \left({T-t_w } \right) M } \right\} \right. \\&\left. -\,cI_p \frac{D\left(p \right) }{\alpha }\left\{ {\frac{1}{\alpha }\left({e^{\alpha \left({t_w -M} \right) }-1} \right) -\left({t_w -M} \right) } \right\} \right] \\&+\,\frac{1}{T}\left\{ {Dp-sD\left({T-t_w } \right) +3cD+pI_e DM} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{2.3} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T-\frac{FY_2 t_r^2 }{2}} \right. -HY_2 \left\{ {t_r \left({t_d -t_r } \right) -\frac{1}{2}\left({t_d^2 -t_r^2 } \right) } \right. \left. \right. \nonumber \\&\left. +\,\frac{1}{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) \right\} \nonumber \\&-\,H\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 -sY_2 \left({T-t_w } \right) ^{2}/2+sD\left(p \right) \left({T-t_w } \right) Y_3 \nonumber \\&-\,cY_2 \left\{ {t_r -\left({T-t_w } \right) } \right\} \nonumber \\&+\,cD\left(p \right) Y_3 +pI_e Y_2 M\left\{ {M/2+\left({T-t_w } \right) } \right\} -pI_e D\left(p \right) MY_3 \nonumber \\&\left. { -\,cI_p \frac{Y_2 }{\alpha }\left\{ {\frac{1}{\alpha }\left({e^{\alpha \left({t_w -M} \right) }-1} \right) -\left({t_w -M} \right) } \right\} -cI_p \frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -M} \right) }-1} \right) Y_3 } \right] =0\nonumber \\ \end{aligned}$$

(42)

$$\begin{aligned} \frac{\partial \textit{TP}_{2.4} }{\partial t_r }= & {} \frac{-D\left(p \right) }{T}\left[{Ft_r +H\left\{ {\left({t_d -t_r } \right) +\frac{1}{\alpha }\left({1-e^{\alpha \left({t_w -t_d } \right) }} \right) Y_1 } \right\} } \right. -s\left({T-t_w } \right) Y_1\\&-\,c\left({e^{\beta \left({t_r -t_d } \right) }+Y_1 } \right) +c\left({1+Y_1 } \right) \\&\left. { -\,pI_e MY_1 } \right] -\frac{HW}{T}=0 \\ \frac{\partial \textit{TP}_{2.4} }{\partial T}= & {} -\frac{1}{T^{2}}\left[{D\left(p \right) pT-A-\frac{FD\left(p \right) t_r^2 }{2}} \right. -H\left\{ Wt_r +D\left(p \right) t_r \left({t_d -t_r } \right) \right. \\&\left. -\,\frac{D\left(p \right) }{2}\left({t_d^2 -t_r^2 } \right) \right. \\&\left. {+\,\frac{D\left(p \right) }{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left(

{t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) } \right\} -sD\left(p \right) \left({T-t_w } \right) ^{2}/2\\&-\,c\left\{ {W+D\left(p \right) t_r -D\left(p \right) \left({T-t_w } \right) } \right\} \\&\left. {+\,\left\{ {\left({pI_e D\left(p \right) t_w^2 } \right) /2+pI_e D\left(p \right) t_w \left({M-t_w } \right) +pI_e D\left(p \right) \left({T-t_w } \right) M} \right\} } \right] \\&+\,\frac{1}{T}\left\{ {Dp-sD\left({T-t_w } \right) +2cD+pI_e DM} \right\} =0 \end{aligned}$$

$$\begin{aligned} \frac{\partial \textit{TP}_{2.4} }{\partial p}= & {} \frac{1}{T}\left[{\left({1-e} \right) D\left(p \right) T-\frac{FY_2 t_r^2 }{2}} \right. \nonumber \\&-\,HY_2 \left\{ {t_r \left({t_d -t_r } \right) -\frac{1}{2}\left({t_d^2 -t_r^2 } \right) } \right. \left. {+\frac{1}{\alpha }\left({\frac{1}{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) -\left({t_w -t_d } \right) } \right) } \right\} \nonumber \\&-\,H\frac{D\left(p \right) }{\alpha }\left({e^{\alpha \left({t_w -t_d } \right) }-1} \right) Y_3 -sY_2 \left({T-t_w } \right) ^{2}/2+sD\left(p \right) \left({T-t_w } \right) Y_3 \nonumber \\&-\,2cY_2\left\{ {t_r -\left({T-t_w } \right) } \right\} -2cD\left(p \right) Y_3 \nonumber \\&-\,cY_2 T\left. +pI_e Y_2 \left\{ {\left({pI_e D\left(p \right) t_w^2 } \right) /2+t_w \left({M-t_w } \right) +\left({T-t_w } \right) M} \right\} \right. \nonumber \\&\left. -\,pI_e D\left(p \right) t_w Y_3 \right] =0 \end{aligned}$$

(43)

where

$$\begin{aligned} X_1= & {} \frac{D\left(p \right) }{D\left(p \right) +\alpha We^{\alpha \left({t_d -t_r } \right) }}, \quad X_2 = Y_2 = \frac{\partial D\left(p \right) }{\partial p}= -ekp^{-\left({e+1} \right) }, \\ X_3= & {} \frac{-D\left(p \right) We^{\alpha \left({t_d -t_r } \right) }}{eX_2 \left({D\left(p \right) +\alpha We^{\alpha \left({t_d -t_r } \right) }} \right) }\\ Y_1= & {} \frac{D\left(p \right) }{\left\{ {D\left(p \right) +\alpha \left({W+D\left(p \right) \left({t_r -t_d } \right) } \right) } \right\} }, \;and\; \\ Y_3= & {} \frac{-WY_2 }{D\left(p \right) \left\{ {D\left(p \right) +\alpha \left({W+D\left(p \right) \left({t_r -t_d } \right) } \right) } \right\} } \end{aligned}$$

Rights and permissions

Reprints and permissions

About this article

Cite this article

Jaggi, C.K., Tiwari, S. & Goel, S.K. Credit financing in economic ordering policies for non-instantaneous deteriorating items with price dependent demand and two storage facilities. *Ann Oper Res* **248**, 253–280 (2017). https://doi.org/10.1007/s10479-016-2179-3

Published
05 April 2016

Issue Date
January 2017

DOI
https://doi.org/10.1007/s10479-016-2179-3

Keywords

Inventory
Non-instantaneous deterioration
Two-warehouse
Permissible delay
Shortages