Financing the newsvendor: raising the loan limit by insurance contract

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Abstract

Banks often control their risks to be below a risk limit through setting a loan limit and the capital constrained newsvendor can make the loan limit increase by buying a guarantee insurance policy. This paper examines the impact of a bank’s risk limit, interest rate setting and initial capital on the newsvendor’s financial and ordering decisions with deductible insurance contract. In the perfectly competitive bank market, it is shown that the newsvendor will restore his profit to the optimal level without capital constraint by buying a full insurance. In the regulated monopolistic bank market, the newsvendor only buys insurance when both his initial capital and the bank’s risk limit are low; for a poorer newsvendor, the bank should require a lower risk limit to force the newsvendor to buy insurance, not just to meet regulatory requirements. It is also shown that the insurance is more useful for a poor newsvendor and a more risk-averse bank.

References

Anderson MH, Prezas AP (1998) The interaction of investment and financing decisions under moral hazard. Int Rev Econ Financ 7(4):379–392

Article Google Scholar

Buzacott JA, Zhang RQ (2004) Inventory management with asset-based financing. Manag Sci 50(9):1274–1292

Article Google Scholar

Cai G, Chen X, Xiao Z (2014) The roles of bank and trade credits: theoretical analysis and empirical evidence. Prod Oper Manag 23(4):583–598

Article Google Scholar

Caillaud B, Dionne G, Jullien B (2000) Corporate insurance with optimal financial contracting. Econ Theor 16(1):77–105

Article Google Scholar

Chen X, Cai G, Song J (2019) The cash flow advantages of 3PLs as supply chain orchestrators. Manuf Serv Oper Manag 21(2):435–451

Article Google Scholar

Dada M, Hu QH (2008) Financing newsvendor inventory. Oper Res Lett 36:569–573

Article Google Scholar

Deng S, Gu C, Cai G et al (2018) Financing multiple heterogeneous suppliers in assembly systems: Buyer finance vs. bank finance. Manuf Serv Oper Manag 20(1):53–69

Article Google Scholar

Devalkar SK, Krishnan H (2019) The impact of working capital financing costs on the efficiency of trade credit. Prod Oper Manag 28(4):878–889

Article Google Scholar

Dong L, Tomlin B (2012) Managing disruption risk: the interplay between operations and insurance. Manag Sci 58(10):1898–1915

Article Google Scholar

Dotan A, Ravid SA (1987) On the interaction of real and financial decisions of the firm under uncertainty. J Financ 40(2):501–507

Article Google Scholar

Gollier C (1996) Deductible insurance and production: a comment. Insur Math Econ 19:55–59

Article Google Scholar

Jing B, Chen X, Cai G (2012) Equilibrium financing in a distribution channel with capital constraint. Prod Oper Manag 21(6):1090–1101

Article Google Scholar

Kouvelis P, Zhao W (2011) The newsvendor problem and price-only contract when bankruptcy costs exist. Prod Oper Manag 20(6):921–936

Article Google Scholar

Kouvelis P, Zhao W (2012) Financing the newsvendor: Supplier vs. bank, and the structure of optimal trade credit contracts. Oper Res 60(3):566–580

Article Google Scholar

Lai G, Debo LG, Sycara K (2009) Sharing inventory risk in supply chain: the implication of financial constraint. Omega 37(4):811–825

Article Google Scholar

Lariviere M, Porteus E (2001) Selling to the newsvendor: an analysis of price-only contracts. Manuf Serv Oper Manag 3(4):293–305

Article Google Scholar

Machnes Y (1995) Deductible insurance and production. Insur Math Econ 17:119–123

Article Google Scholar

Machnes Y, Wong K (2003) A note on deductible insurance and production. Geneva Pap Risk Insur Theor 28:73–80

Article Google Scholar

Modigliani F, Miller M (1958) The cost of capital, corporation finance and the theory of investment. Am Econ Rev 48(3):261–297

Google Scholar

Myers SC (1974) Interaction of corporate financing and investment decisions–implications for capital budgeting. J Financ 29:1–25

Article Google Scholar

Myers SC (1977) Determinants of corporate borrowing. J Financ Econ 5:147–176

Article Google Scholar

Reindorp M, Tanrisever F, Lange A (2018) Purchase order financing: credit, commitment, and supply chain consequences. Oper Res 66(5):1287–1303

Article Google Scholar

Serpa J, Krishnan H (2017) The strategic role of business insurance. Manag Sci

63(2):384–404

Article Google Scholar

SGI Website (2019). https://www.sgic.co.kr/chp/iutf/en/html/Products.mvc?tm=2&sm=1

Stiglitz JE (1972) Some aspects of the pure theory of corporate finance: bankruptcies and take-overs. Bell J Econ Manag Sci 3:458–482

Article Google Scholar

Tang CS, Yang SA, Wu J (2018) Sourcing from suppliers with financial constraints and performance risk. Manuf Serv Oper Manag 20(1):70–84

Article Google Scholar

Tunca TI, Zhu W (2018) Buyer intermediation in supplier finance. Manag Sci 64(12):5631–5650

Article Google Scholar

Xu X, Birge JR (2008) Operational decisions, capital structure, and managerial compensation: a news vendor perspective. Eng Econ 53:173–196

Article Google Scholar

Yang SA, Bakshi N, Chen C (2019) Trade credit insurance: operational value and contract choice. Working paper, London Business School

Zhang Y, Huang S, Yang SA (2019) Financing inventory under bank capital regulation and seller orchestration. Working paper, Tsinghua University

Zhou W, Lin T, Cai G (2017) Guarantor financing in a four-party supply chain with leadership influence. Working paper, Santa Clara University

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Publisher's Note

Appendix

Proof of Proposition 1

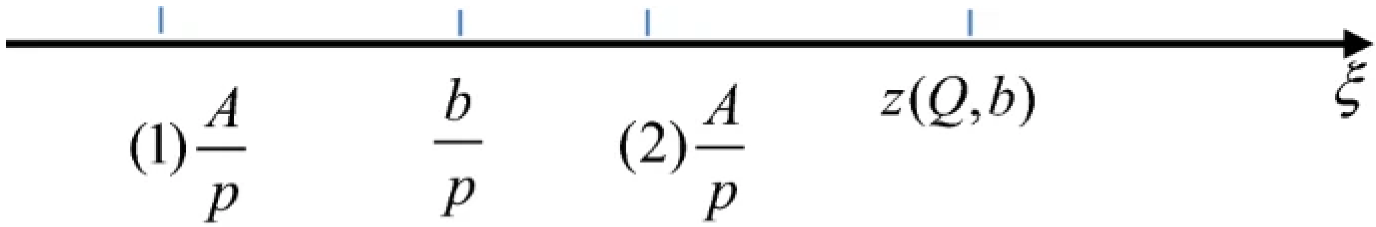
Define $A = (1 - \alpha)(wQ + m(b) - B)$, we have $A < pz(Q,b)$ always holds. If the newsvendor borrows money, the bank's risk is as follows (Fig. 8):

$$\begin{aligned} & \Pr \{ Y(Q,b) \le A\} \\ & \quad = \Pr \left\{ {b < A\left| {0 \le \xi < \frac{b}{p}} \right.} \right\}\Pr \left({0 \le \xi < \frac{b}{p}} \right) \\ & \quad \quad + \,\Pr \left\{ {p\xi \le A\left| {\frac{b}{p} \le \xi < z(Q,b)} \right.} \right\}\Pr \left({\frac{b}{p} \le \xi < z(Q,b)} \right) \\ & \quad \quad + \,\Pr \{ (wQ + m(b) - B)(1 + r) \le A|\xi \ge z(Q,b)\} \Pr (\xi \ge z(Q,b)) \\ & \quad = \,\Pr \left\{ {b < A\left| {0 \le \xi < \frac{b}{p}} \right.} \right\}F\left({\frac{b}{p}} \right) + \Pr \left\{ {p\xi \le A\left| {\frac{b}{p} \le \xi < z(Q,b)} \right.} \right\}\left[{F(z(Q,b)) - F\left({\frac{b}{p}} \right)} \right] \\ \end{aligned}$$

1. If $\frac{A}{p} < \frac{b}{p}$, that is, $Q < \frac{B}{w} + \frac{b}{w(1 - \alpha)} - \int_{0}^{{\frac{b}{p}}} {\frac{p}{w}F(x)dx} \equiv Q_{2} (b)$, we have $\Pr \{ Y(Q,b) \le A\} = 0$, which means the bank can control risk all the time.

2. If $\frac{A}{p} \ge \frac{b}{p}$, that is, $Q \ge \frac{B}{w} + \frac{b}{w(1 - \alpha)} - \int_{0}^{{\frac{b}{p}}} {\frac{p}{w}F(x)dx} \equiv Q_{2} (b)$, we have $\Pr \{ Y(Q,b) \le A\} = F\left({\frac{b}{p}} \right) + \Pr \left({\frac{b}{p} < \xi < \frac{A}{p}} \right) = F\left({\frac{A}{p}} \right)$. Thus, $F\left({\frac{A}{p}} \right) \le \beta$, that is, $Q \le \frac{{B{ + }L(\beta)}}{w} - \int_{0}^{{\frac{b}{p}}} {\frac{p}{w}F(x)dx} \equiv Q_{3} (b)$, must hold to control the bank's risk.

Fig. 8



The different location of $\frac{A}{p}$

It is easy to see that $Q_{2} (b)$ increases with b, $Q_{3} (b)$ decreases with b. Because $Q_{2} (0) < Q_{3} (0)$, there exists a unique insurance coverage level $b_{0} = pF^{ - 1} (\beta)$, such that the newsvendor’s order limit is $Q_{3} (b)$ for $b \le b_{0}$ and $Q_{2} (b)$ for $b > b_{0}$.

Proof of Lemma 1

Because $Q_{2} (b_{1}) = \frac{{B{ + }L(\beta)}}{w}$, we have (10) holds. Using the Implicit Function Theorem of (10), we have

$$\frac{{db_{1} }}{d\beta } = \frac{1}{{f(F^{ - 1} (\beta))\left[{\frac{1}{p(1 - \alpha)} - \frac{1}{p}F\left({\frac{{b_{1} }}{p}} \right)} \right]}} > 0$$

Thus, b_{1} increases with β.

Proof of Proposition 2

When the newsvendor borrows money from the bank, the bank’s risk can be defined as

$$\begin{aligned} & \Pr \{ Y(Q) \le (1 - \alpha)(wQ - B)\} \\ & \quad = \,\Pr \{ p\xi \le (1 - \alpha)(wQ - B)|0 \le \xi < z(Q)\} \Pr (0 \le \xi < z(Q)) \\ & \quad \quad + \,\Pr \{ (wQ - B)(1 + r(Q)) \le (1 - \alpha)(wQ - B)|\xi \ge z(Q)\} \Pr (\xi \ge z(Q)) \\ & \quad = \,\Pr \left\{ {\left. {\xi \le \frac{(1 - \alpha)(wQ - B)}{p}} \right|0 \le \xi < z(Q)} \right\}F(z(Q)) \\ & \quad = \,F\left({\frac{(1 - \alpha)(wQ - B)}{p}} \right) \\ \end{aligned}$$

Thus, the bank will set the loan limit to let the newsvendor’s order limit equal Q_{1}. Because Q_{1} increases with B, the bank’s risk control does not work if $L(\beta) \ge wQ^{N}$. If $L(\beta) < wQ^{N}$, the newsvendor’s optimal order quantity is $Q^{*} = \hbox{min} (Q_{1} ,Q^{N})$.

Proof of Lemma 2

Define

$$M(Q,b) = p\left[{z(Q,b) - \int_{{\frac{b}{p}}}^{z(Q,b)} {F(x)dx} } \right] - [wQ + m(b) - B]$$

Based on Implicit Function Theorem, we have

$$\frac{\partial r(Q,b)}{\partial b} = - \frac{\partial M(Q,b)/\partial b}{\partial M(Q,b)/\partial r} = - \frac{{F\left({\frac{b}{p}} \right)(1 + r(Q,b))}}{wQ + m(b) - B} < 0$$

$$\frac{\partial z(Q,b)}{\partial b} = \frac{1 + r(Q,b)}{p}\frac{dm(b)}{db} + \frac{wQ + m(b) - B}{p}\frac{\partial r(Q,b)}{\partial b} = 0$$

which implies the results.

Proof of Proposition 4

If the newsvendor borrows money, the bank's risk is the same as that in the perfectly competitive bank market. Thus, the bank will set the loan limit such that the newsvendor's order limit is Q_{1}. Using the Implicit Function Theorem of (14), we have

$$\begin{aligned} \frac{{dQ^{LO} }}{dB} & = \frac{{\frac{{w(1 + r)^{2} }}{p}f(z(Q^{LO}))}}{{\frac{{w^{2} (1 + r)^{2} }}{p}f(z(Q^{LO})) - pf(Q^{LO})}} \\ & = \frac{{w(1 + r)^{2} f(z(Q^{LO}))}}{{p[1 - F(Q^{LO})][w(1 + r)h(z(Q^{LO})) - ph(Q^{LO})]}} < 0 \\ \end{aligned}$$

Thus, Q^{LO} decreases with B. When $B = wQ^{M}$, $Q^{LO} = Q^{M}$, so $Q^{LO} > Q^{M}$ for $B < wQ^{M}$.

Because Q_{1} increases with B, the bank's risk control constraint does not work if $L(\beta) \ge wQ_{ 0}^{LO}$. If $L(\beta) < wQ_{ 0}^{LO}$, when $B = wQ^{M}$, $Q^{LO} = Q^{M} < Q_{1}$, so there exists a unique $B_{0} \in (0,wQ^{M})$ satisfies $p[1 - F(Q_{1} (B_{0}))] = w(1 + r)[1 - F(z(Q_{1} (B_{0})))]$ such that $Q^{LO} > Q_{1}$ for $B < B_{0}$ and $Q^{LO} < Q_{1}$ for

$B > B_{0}$. Besides, the conclusion that B_{0} decreases with β can be easily obtained from Lemma 5.

Proof of Lemma 3

The first-order derivatives of $\varPi_{M}^{LI} (Q,b)$ with respect to b and Q are

$$\frac{{\partial \varPi_{M}^{LI} (Q,b)}}{\partial b} = - [1 - F(z(Q,b))]F\left({\frac{b}{p}} \right)(1 + r) < 0$$

$$\frac{{\partial \varPi_{M}^{LI} (Q,b)}}{\partial Q} = p[1 - F(Q)] - w(1 + r)[1 - F(z(Q,b))]$$

Given b, $\frac{{\partial \varPi_{M}^{LI} (Q,b)}}{\partial Q}$ decreases with Q at $Q = Q^{LI} (b)$.

Using the Implicit Function Theorem of (17), we have

$$\frac{{dQ^{LI} (b)}}{db} = \frac{{w(1 + r)^{2} f(z(Q^{LI} (b),b))F\left({\frac{b}{p}} \right)}}{{p[1 - F(Q^{LI} (b))][ph(Q^{LI} (b)) - w(1 + r)h(z(Q^{LI} (b),b))]}} > 0$$

Thus, $Q^{LI} (b)$ increases with b.

Proof of Lemma 4

For $B < B_{0}$,

$$\varPi_{M}^{LI} (Q_{3} (b),b) = p\left[{Q_{3} (b) - \frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)} \right] - \int_{{\frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)}}^{{Q_{3} (b)}} {pF(x)dx} - B$$

$$\varPi_{M}^{LI} (Q^{LI} (b),b) = p[Q^{LI} (b) - z(Q^{LI} (b),b)] - \int_{{z(Q^{LI} (b),b)}}^{{Q^{LI} (b)}} {pF(x)dx} - B$$

$$\varPi_{M}^{LI} (Q_{2} (b),b) = p\left[{Q_{2} (b) - \frac{b(1 + r)}{p(1 -

\alpha)}} \right] - \int_{{\frac{b(1 + r)}{p(1 - \alpha)}}}^{{Q_{2} (b)}} {pF(x)dx} - B$$

Because

$$\frac{{d\varPi_{M}^{LI} (Q_{3} (b),b)}}{db} = - p[1 - F(Q_{3} (b))]\frac{1}{w}F\left({\frac{b}{p}} \right) < 0,$$

$$\frac{{d\varPi_{M}^{LI} (Q^{S} (b),b)}}{db} = \frac{{\partial \varPi_{M}^{LI} (Q^{LI} (b),b)}}{{\partial Q^{LI} (b)}}\frac{{dQ^{LI} (b)}}{db} + \frac{{\partial \varPi_{M}^{LI} (Q^{LI} (b),b)}}{\partial b} = \frac{{\partial \varPi_{M}^{LI} (Q^{LI} (b),b)}}{\partial b} < 0$$

Both \(\varPi_{M}^{LI} (Q_{3} (b),b)\) and \(\varPi_{M}^{LI} (Q^{LI} (b),b)\) decrease with \(b\).

Because\(\frac{{d\varPi_{M}^{LI} (Q_{2} (b),b)}}{db} = p[1 - F(Q_{2} (b))]\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{b}{p}} \right)} \right] - \left[{1 - F\left({\frac{b(1 + r)}{p(1 - \alpha)}} \right)} \right]\frac{1 + r}{1 - \alpha }\), and \(b^{S}\) satisfies the first-order optimality condition,

$$\begin{aligned} & \left. {\frac{{d^{2} \varPi_{M}^{LI} Q_{2} (b),b)}}{{db^{2} }}} \right|_{{b = b^{S} }} = - pf(Q_{2} (b^{S}))\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{b}{p}} \right)} \right]^{2} - [1 - F(Q_{2} (b^{S}))]\frac{1}{w}f\left({\frac{{b^{S} }}{p}} \right) + pf(z(Q_{2} (b^{S}),b))\frac{{(1 + r)^{2} }}{{p^{2} (1 - \alpha)^{2} }} \\ & \quad = - p[1 - F(Q_{2} (b^{S}))]\left\{ {\frac{{f(Q_{2} (b^{S}))}}{{1 - F(Q_{2} (b^{S}))}}\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right]^{2} - \frac{{f(z(Q_{2} (b^{S}),b))}}{{1 - F(Q_{2} (b^{S}))}}\frac{{(1 + r)^{2} }}{{p^{2} (1 - \alpha)^{2} }}} \right\} - [1 - F(Q_{2} (b^{S}))]\frac{1}{w}f\left({\frac{b}{p}} \right) \\ & \quad = - p[1 - F(Q_{2} (b^{S}))]\left\{ {h(Q_{2} (b^{S}))\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right]^{2} } \right. \\ & \quad \quad \left. { - \,h(z(Q_{2} (b^{S}),b))\frac{(1 + r)}{p(1 - \alpha)}\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right]} \right\} - [1 -

F(Q_{2} (b^{S}))]\frac{1}{w}f\left({\frac{b}{p}} \right) \\ & \quad = - p[1 - F(Q_{2} (b^{S}))]\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right] \\ & \quad \quad \quad \left\{ {h(Q_{2} (b^{S}))\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right] - h(z(Q_{2} (b^{S}),b))\frac{1 + r}{p(1 - \alpha)}} \right\} - [1 - F(Q_{2} (b^{S}))]\frac{1}{w}f\left({\frac{b}{p}} \right) \\ \end{aligned}$$

From (21), we can see that $\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right) \ge \frac{1 + r}{p(1 - \alpha)}$ holds since $Q_{2} (b^{S}) \ge z(Q_{2} (b^{S}),b^{S}) = \frac{{b^{S} (1 + r)}}{p(1 - \alpha)}$. Thus, $\left. {\frac{{d^{2} \varPi_{M}^{LI} (Q_{2} (b),b)}}{{db^{2} }}} \right|_{{b = b^{S} }} < 0$ which means $\varPi_{M}^{LI} (Q_{2} (b),b)$ is concave and obtains the maximal value in $b = b^{S}$.

Using the Implicit Function Theorem of (21), we have

$$\frac{{db^{S} }}{dB} = \frac{{ph(Q_{2} (b^{S}))\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right]}}{{ - f\left({\frac{{b^{S} }}{p}} \right) + \left[{\frac{1}{1 - \alpha } - F\left({\frac{{b^{S} }}{p}} \right)} \right]\left\{ {\frac{1 + r}{1 - \alpha }h\left({\frac{{b^{S} (1 + r)}}{p(1 - \alpha)}} \right) - \left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right)} \right]ph(Q_{2} (b^{S}))} \right\}}}$$

Because $Q_{2} (b^{S}) \ge z(Q_{2} (b^{S}),b^{S}) = \frac{{b^{S} (1 + r)}}{p(1 - \alpha)}$ and $\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b^{S} }}{p}} \right) \ge \frac{1 + r}{p(1 - \alpha)}$, $\frac{{db^{S} }}{dB} < 0$ always holds.

Define $H(b) = \frac{{d\varPi_{M}^{LI} (Q_{2} (b),b)}}{db}$ and we have

$$\begin{aligned} H(b_{2}) & = p[1 - F(Q_{2} (b_{2}))]\left[{\frac{1}{w(1 - \alpha)} - \frac{1}{w}F\left({\frac{{b_{2} }}{p}} \right)} \right] - [1 - z(Q_{2} (b_{2}),b_{2})]\frac{1 + r}{1 - \alpha } \\ & = (1 + r)\left[{1 - F\left(

{\frac{{b_{2} (1 + r)}}{p(1 - \alpha)}} \right)} \right]\left[{\frac{1}{1 - \alpha } - F\left({\frac{{b_{2} }}{p}} \right)} \right] - \left[{1 - F\left({\frac{{b_{2} (1 + r)}}{p(1 - \alpha)}} \right)} \right]\frac{1 + r}{1 - \alpha } \\ & = - \left[{1 - F\left({\frac{{b_{2} (1 + r)}}{p(1 - \alpha)}} \right)} \right](1 + r)F\left({\frac{{b_{2} }}{p}} \right) < 0 \\ \end{aligned}$$

Thus, \(b^{S} < b_{2}\) holds. Similarly, we have \(H(0) = p\left[{1 - F\left({\frac{B}{w}} \right)} \right]\frac{1}{w(1 - \alpha)} - \frac{1 + r}{1 - \alpha }\). When \(B < B_{0}\), we have \(B < wQ^{M}\), \(H(0) > 0\). Thus, \(b^{S} > 0\) holds.

Proof of Lemma 5

The first-order derivative of \(\varPi_{M}^{LO} (Q_{ 1})\) with respect to \(\beta\) is

$$\frac{{d\varPi_{M}^{LO} (Q_{ 1})}}{d\beta } = \frac{p}{{(1 - \alpha)f(F^{ - 1} (\beta))}}G(\beta)$$

where

$$G(\beta) = \frac{p}{w}[1 - F(Q_{1})] - (1 + r)\left[{1 - F\left({\frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)} \right)} \right]$$

\(\beta_{1}\) is the solution of \(G(\beta) = 0\). Because

$$\begin{aligned} G^{{\prime }} (\beta)|_{{\beta = \beta_{1} }} & = \frac{1}{{(1 - \alpha)f(F^{ - 1} (\beta))}}\left\{ { - \frac{{p^{2} }}{{w^{2} }}f(Q_{1}) + (1 + r)^{2} f\left({\frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)} \right)} \right\} \\ & = \frac{{(1 + r)[1 - F(Q_{1})]}}{{(1 - \alpha)f(F^{ - 1} (\beta))}}\left\{ {(1 + r)h(Q_{1}) - \frac{p}{w}h\left({\frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)} \right)} \right\} < 0 \\ \end{aligned}$$

we have

$$\frac{{d\varPi_{M}^{LO} (Q_{1})}}{d\beta }|_{{\beta = \beta_{1} }} = \left[

{\frac{p}{{(1 - \alpha)f(F^{ - 1} (\beta_{1}))}}} \right]^{{\prime }} G(\beta_{1}) + \frac{p}{{(1 - \alpha)f(F^{ - 1} (\beta_{1}))}}G^{{\prime }} (\beta_{1}) < 0$$

Thus, $\varPi_{M}^{LO} (Q_{1})$ obtains the maximal value at $\beta = \beta_{1}$. Using the Implicit Function Theorem of (22), we have

$$\frac{{d\beta_{1} }}{dB} = \frac{{\frac{1}{w}h(Q_{1})}}{{\frac{1}{{(1 - \alpha)f(F^{ - 1} (\beta_{1}))}}\left\{ {(1 + r)h\left({\frac{1 + r}{1 - \alpha }F^{ - 1} (\beta)} \right) - \frac{p}{w}h(Q_{1})} \right\}}} < 0$$

Thus, β_{1} decreases with B. When $B = wQ^{M}$, $\beta_{1} = 0$, so $\beta_{1} > 0$ for $B < B_{0}$ since $B_{0} < wQ^{M}$.

Proof of Proposition 7

When $\beta = 0$, let

$$T(B) = \varPi_{M}^{LO} (Q_{1}) - \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S}) = pS\left({\frac{B}{w}} \right) - B - \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$$

and we have

$$\begin{aligned} \frac{dT(B)}{dB} & = \frac{p}{w}\left[{1 - F\left({\frac{B}{w}} \right)} \right] - 1 - \frac{{\partial \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})}}{{\partial b^{S} }}\frac{{db^{S} }}{dB} - \frac{p}{w}[1 - F(Q_{2} (b^{S}))] + 1 \\ & = \frac{p}{w}\left[{F(Q_{2} (b^{S})) - F\left({\frac{B}{w}} \right)} \right] > 0 \\ \end{aligned}$$

Thus, $T(B)$ increases with B. When $B = wQ^{M}$, $b^{S} = 0$, so $T(B) = 0$. Thus, $T(B) < 0$ always holds for $B < B_{0}$ since $T(0) < 0$ and $B_{0} \le wQ^{M}$. That is to say, $\varPi_{M}^{LO} (Q_{1}) < \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$ holds for $\beta = 0$.

When $\beta = \beta_{1}$, i.e. $B = B_{0}$, $Q_{ 1} = Q^{LO}$, so we have

$$\varPi_{M}^{LO} (Q_{1}) = \varPi_{M}^{LO} (Q^{LO}) > \varPi_{M}^{LO} (Q_{2} (b^{S})) > \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$$

Because $\varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$ is independent of β and $\varPi_{M}^{LO} (Q_{1})$ increases with β for $B < B_{0}$, there exist a unique value $\beta_{2} \in (0,\beta_{1})$, such that $\varPi_{M}^{LO} (Q_{1}) < \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$ for $\beta < \beta_{2}$ and $\varPi_{M}^{LO} (Q_{1}) > \varPi_{M}^{LI} (Q_{2} (b^{S}),b^{S})$ for $\beta < \beta_{2}$.

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