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PRICING IN AN INCOMPLETE MARKET WITH AN AFFINE TERM STRUCTURE

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Abstract

We apply the principle of equivalent utility to calculate the indifference price of the writer of a contingent claim in an incomplete market. To recognize the long-term nature of many such claims, we allow the short rate to be random in such a way that the term structure is affine. We also consider a general diffusion process for the risky stock (index) in our market. In a complete market setting, the resulting indifference price is the same as the one obtained by no-arbitrage arguments. We also show how to compute indifference prices for two types of contingent claims in an incomplete market, in the case for which the utility function is exponential. The first is a catastrophe risk bond that pays a fixed amount at a given time if a catastrophe does *not* occur before that time. The second is equity-indexed term life insurance which pays a death benefit that is a function of the short rate and stock price at the random time of the death of the insured. Because we assume that the occurrence of the catastrophe or the death of the insured is independent of the financial market, the markets for the catastrophe risk bond and the equity-indexed life insurance are incomplete.

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