

# PRICING IN AN INCOMPLETE MARKET WITH AN AFFINE TERM STRUCTURE

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## Abstract

We apply the principle of equivalent utility to calculate the indifference price of the writer of a contingent claim in an incomplete market. To recognize the long-term nature of many such claims, we allow the short rate to be random in such a way that the term structure is affine. We also consider a general diffusion process for the risky stock (index) in our market. In a complete market setting, the resulting indifference price is the same as the one obtained by no-arbitrage arguments. We also show how to compute indifference prices for two types of contingent claims in an incomplete market, in the case for which the utility function is exponential. The first is a catastrophe risk bond that pays a fixed amount at a given time if a catastrophe does *not* occur before that time. The second is equity-indexed term life insurance which pays a death benefit that is a function of the short rate and stock price at the random time of the death of the insured. Because we assume that the occurrence of the catastrophe or the death of the insured is independent of the financial market, the markets for the catastrophe risk bond and the equity-indexed life insurance are incomplete.

## REFERENCES

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---

Bacinello, A. R., and F. Ortu (1994): Single and Periodic Premiums for Guaranteed Equity-Linked Life Insurance under Interest-Rate Risk: The “Lognormal + Vasicek” Case; in *Financial Modeling*, L. Peccati and M. Viren, eds. Heidelberg, Germany, Physica-Verlag, 1–25.

[Google Scholar](#) 

---

Barles, G., and H. M. Soner (1998): Option Pricing with Transaction Costs and a Nonlinear Black-Scholes Equation, *Finance Stoch.* 2, 369–397.

[Google Scholar](#) 

---

Björk, T. (1998): *Arbitrage Theory in Continuous Time*. New York : Oxford University Press.

[Google Scholar](#) 

---

Black, F., and M. Scholes (1973): The Pricing of Options and Corporate Liabilities, *J. Polit. Econ.* 81, 637–654.

[Web of Science®](#)  | [Google Scholar](#) 

---

Bowers, N. L., H. U. Gerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt (1997): *Actuarial Mathematics*, 2nd ed. Schaumburg, IL : Society of Actuaries .

[Google Scholar](#) 

---

Boyle, P. P., and E. S. Schwarz (1977): Equilibrium Prices of Guarantees under Equity-Linked Contracts, *J. Risk & Ins.* 44, 639–660.

[Web of Science®](#)  | [Google Scholar](#) 

---

Brennan, M. J., and E. S. Schwartz (1976): The Pricing of Equity-Linked Life Insurance Policies with an Asset Value Guarantee, *J. Financial Econ.* 3, 195–213.

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---

Constantinides, G. M., and T. Zariphopoulou (1999): Bounds on Prices of Contingent Claims in an Intertemporal Economy with Proportional Transaction Costs and General Preferences. *Finance Stoch.* 3, 345–369.

[Google Scholar](#)

---

Constantinides, G. M., and T. Zariphopoulou (2001): Bounds on Derivative Prices in an Intertemporal Setting with Proportional Costs and Multiple Securities, *Math. Finance* 11, 331–346.

[Web of Science®](#) | [Google Scholar](#)

---

Cox, S. H., and H. W. Pedersen (2000): Catastrophe Risk Bonds, *N. Am. Actuarial J.* 4(4), 56–82.

[Google Scholar](#)

---

Cvitanic, J., P. Huyen, and N. Touzi (1999): Super-Replication in Stochastic Volatility Models under Portfolio Constraints, *J. Appl. Prob.* 36(2), 523–545.

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---

Cvitanic, J., and I. Karatzas (1996): Hedging and Portfolio Optimization under Transaction Costs: A Martingale Approach, *Math. Finance* 6, 133–165.

[Google Scholar](#)

---

Davis, M. H. A. (1997): Option Pricing in Incomplete Markets; in *Mathematics of Derivative Securities*, M. A. H. Dempster and S. R. Pliska, eds. Publications of the Newton Institute, Cambridge: Cambridge University Press, 15, 216–226.

[Web of Science®](#) | [Google Scholar](#)

---

Davis, M. H. A., V. G. Panas, and T. Zariphopoulou (1993): European Option Pricing with Transaction Costs, *SIAM J. Control Optim.* 31, 470–493.

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[Google Scholar](#) 

---

Ekern, S., and S.-A. Persson (1996): Exotic Unit-Linked Life Insurance Contracts, *Geneva Papers on Risk & Ins. Theory* 21, 35–63.

[Web of Science®](#)  | [Google Scholar](#) 

---

Gerber, H. U. (1976): A Probabilistic Model for (Life) Contingencies and a Delta-Free Approach to Contingency Reserves, with discussion, *Trans. Soc. Actuar.* 28, 127–148.

[Google Scholar](#) 

---

Heston, S. (1993): A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies* 6, 327–334.

[Web of Science®](#)  | [Google Scholar](#) 

---

Hodges, S. D., and A. Neuberger (1989): Optimal Replication of Contingent Claims under Transaction Costs, *Rev. Futures Mkts* 8, 222–239.

[Google Scholar](#) 

---

Hull, J., and A. White (1987): The Pricing of Options on Assets with Stochastic Volatilities, *Journal of Finance* 42, 281–300.

[Web of Science®](#)  | [Google Scholar](#) 

---

Jouini, E., and H. Kallal (1995): Arbitrage in Securities Markets with Short-Sales Constraints, *Mathematical Finance* 5, 197–232.

[Google Scholar](#) 

---

Karatzas, I., and S. G. Kou (1996): On the Pricing of Contingent Claims under Constraints, *Ann. Appl. Prob.* 6, 321–369.

[Google Scholar](#) 

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[Google Scholar](#) 

---

Leland, H. E. (1985): Option Pricing and Replication with Transactions Costs, *Journal of Finance* **40**, 1283–1301.

[Web of Science®](#)  | [Google Scholar](#) 

---

Moore, K. S., and V. R. Young (2002): Pricing Equity-Linked Pure Endowments via the Principle of Equivalent Utility. Working paper, Department of Mathematics, University of Michigan .

[Google Scholar](#) 

---

Musiela, M., and T. Zariphopoulou (2002): Indifference Prices and Related Measures. Working paper, University of Texas at Austin .

[Google Scholar](#) 

---

Nielsen, J. A., and K. Sandmann (1995): Equity-Linked Life Insurance: A Model with Stochastic Interest Rates, *Insurance: Math. & Econ.* **16**, 225–253.

[Web of Science®](#)  | [Google Scholar](#) 

---

Persson, S.-A. (1993): Valuation of a Multistate Life Insurance Contract with Random Benefits, *Scand. J. Mgmt.* **9**, 573–586.

[Google Scholar](#) 

---


Pratt, J. W. (1964): Risk Aversion in the Small and in the Large, *Econometrica* **32**, 122–136.

[Web of Science®](#)  | [Google Scholar](#) 

---

Renault, E., and N. Touzi (1990): Option Hedging and Implied Volatilities in a Stochastic Volatility Model, *Mathematical Finance* **6**, 279–302.

[Google Scholar](#) 

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Tiong, S. (2000): Valuing Equity-Indexed Annuities, with discussion, *N. Am. Actuarial J.* 4, 149–170.

[Google Scholar](#) 

Wang, S. (1995): Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms, *Insurance: Math. & Econ.* 17, 43–54.

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Wilmott, P., S. Howison, and J. Dewynne (1995): *The Mathematics of Financial Derivatives: A Student Introduction*. Cambridge : Cambridge University Press.

[Google Scholar](#) 

Young, V. R. (2003): Equity-Indexed Life Insurance: Pricing and Reserving Using the Principle of Equivalent Utility, *N. Am. Actuarial J.* 7(1), 68–86.

[Google Scholar](#) 


Young, V. R., and T. Zariphopoulou (2002a): Pricing Dynamic Insurance Risks Using the Principle of Equivalent Utility, *Scand. Actuarial J.* 2002(4), 246–279.

[Google Scholar](#) 

Young, V. R., and T. Zariphopoulou (2002b): Pricing Insurance via Stochastic Control: Optimal Consumption and Terminal Wealth. Working paper, School of Business, University of Wisconsin-Madison .

[Google Scholar](#) 

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