Mathematical Finance / Volume 14, Issue 3 / pp. 359-381

PRICING IN AN INCOMPLETE MARKET WITH AN AFFINE TERM STRUCTURE

Virginia R. Young

First published: 18 June 2004
https://doi.org/10.1111/j.0960-1627.2004.00195.x
Citations: 39

Address correspondence to the author at Department of Mathematics, University of Michigan, 525 E. University, Ann Arbor, MI 48109-1109; e-mail: vryoung@umich.edu.

Manuscript received June 2002; final revision received April 2003.

Abstract

We apply the principle of equivalent utility to calculate the indifference price of the writer of a contingent claim in an incomplete market. To recognize the long-term nature of many such claims, we allow the short rate to be random in such a way that the term structure is affine. We also consider a general diffusion process for the risky stock (index) in our market. In a complete market setting, the resulting indifference price is the same as the one obtained by no-arbitrage arguments. We also show how to compute indifference prices for two types of contingent claims in an incomplete market, in the case for which the utility function is exponential. The first is a catastrophe risk bond that pays a fixed amount at a given time if a catastrophe does *not* occur before that time. The second is equity-indexed term life insurance which pays a death benefit that is a function of the short rate and stock price at the random time of the death of the insured. Because we assume that the occurrence of the catastrophe or the death of the insured is independent of the financial market, the markets for the catastrophe risk bond and the equity-indexed life insurance are incomplete.

REFERENCES

Aase, K. K., and S.-A. Persson (1994): Pricing of Unit-Linked Life Insurance Policies, *Scand. Actuarial J.* **1994**(1), 26–52.

Google Scholar

Bacinello, A. R., and F. Ortu (1993): Pricing Equity-Linked Life Insurance with Endogenous Minimum Guarantees, *Insurance: Math. & Econ*. **12**, 245–257.

Web of Science® | Google Scholar

Bacinello, A. R., and F. Ortu (1994): Single and Periodic Premiums for Guaranteed Equity-Linked Life Insurance under Interest-Rate Risk: The "Lognormal + Vasicek" Case; in *Financial Modeling*, L. Peccati and M. Viren, eds. Heidelberg , Germany , Physica-Verlag, 1–25.

Google Scholar

Barles, G., and H. M. Soner (1998): Option Pricing with Transaction Costs and a Nonlinear Black-Scholes Equation, *Finance Stoch*. **2**, 369–397.

Google Scholar

Björk, T. (1998): *Arbitrage Theory in Continuous Time*. New York : Oxford University Press.

Google Scholar

Black, F., and M. Scholes (1973): The Pricing of Options and Corporate Liabilities, *J. Polit. Econ*. **81**, 637–654.

Web of Science® | Google Scholar

Bowers, N. L., H. U. Gerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt (1997): *Actuarial Mathematics*, 2nd ed. Schaumburg , IL : Society of Actuaries .

Google Scholar

Boyle, P. P., and E. S. Schwarz (1977): Equilibrium Prices of Guarantees under Equity-Linked Contracts, *J. Risk & Ins*. **44**, 639–660.

Web of Science® | Google Scholar

Brennan, M. J., and E. S. Schwartz (1976): The Pricing of Equity-Linked Life Insurance Policies with an Asset Value Guarantee, *J. Financial Econ*. **3**, 195–213.

Web of Science® | Google Scholar

Brennan, M. J., and E. S. Schwartz (1979): Alternative Investment Strategies for the Issuers of Equity Linked Life Insurance Policies with an Asset Value Guarantee, *J. Business* **52**(1), 63–93.

Web of Science® | Google Scholar

Browne, S. (1995): Optimal Investment Policies for a Firm with a Random Risk Process: Exponential Utility and Minimizing the Probability of Ruin, *Math. Oper. Res*. **20**(4), 937–958.

Web of Science® | Google Scholar

Constantinides, G. M., and T. Zariphopoulou (1999): Bounds on Prices of Contingent Claims in an Intertemporal Economy with Proportional Transaction Costs and General Preferences. *Finance Stoch*. **3**, 345–369.

Google Scholar

Constantinides, G. M., and T. Zariphopoulou (2001): Bounds on Derivative Prices in an Intertemporal Setting with Proportional Costs and Multiple Securities, *Math. Finance* **11**, 331–346.

Web of Science® Google Scholar

Cox, S. H., and H. W. Pedersen (2000): Catastrophe Risk Bonds, *N. Am. Actuarial J*. **4**(4), 56–82.

Google Scholar

Cvitanic, J., P. Huyen, and N. Touzi (1999): Super-Replication in Stochastic Volatility Models under Portfolio Constraints, *J. Appl. Prob*. **36**(2), 523–545.

Web of Science® Google Scholar

Cvitanic, J., and I. Karatzas (1996): Hedging and Portfolio Optimization under Transaction Costs: A Martingale Approach, *Math. Finance* **6**, 133–165.

Google Scholar

Davis, M. H. A. (1997): Option Pricing in Incomplete Markets; in *Mathematics of Derivative Securities*, M. A. H. Dempster and S. R. Pliska, eds. Publications of the Newton Institute , Cambridge : Cambridge University Press, **15**, 216–226.

Web of Science® Google Scholar

Davis, M. H. A., V. G. Panas, and T. Zariphopoulou (1993): European Option Pricing with Transaction Costs, *SIAM J. Control Optim*. **31**, 470–493.

Web of Science® Google Scholar

Davis, M. H. A., and T. Zariphopoulou (1995): American Options and Transaction Fees; in *Mathematical Finance*, M. Davis, D. Duffie, W. Fleming, and S. Shreve, eds. *IMA* , New York : Springer-Verlag, **65**, 47–62.

Google Scholar

Dupire, B. (1994): Pricing with a Smile, *Risk* **7**(1), 18–20.

Google Scholar

Ekern, S., and S.-A. Persson (1996): Exotic Unit-Linked Life Insurance Contracts, *Geneva Papers on Risk & Ins. Theory* **21**, 35–63.

Web of Science® | Google Scholar

Gerber, H. U. (1976): A Probabilistic Model for (Life) Contingencies and a Delta-Free Approach to Contingency Reserves, with discussion, *Trans. Soc. Actuar*. **28**, 127–148.

Google Scholar

Heston, S. (1993): A Closed-Ferm Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options, *Review of Financial Studies* **6**, 327–334.

Web of Science® | Google Scholar

Hodges, S. D., and A. Neuberger (1989): Optimal Replication of Contingent Claims under Transaction Costs, *Rev. Futures Mkts* **8**, 222–239.

Google Scholar

Hull, J., and A. White (1987): The Pricing of Options on Assets with Stochastic Volatilities, *Journal of Finance* **42**, 281–300.

Web of Science® | Google Scholar

Jouini, E., and H. Kallal (1995): Arbitrage in Securities Markets with Short-Sales Constraints, *Mathematical Finance* **5**, 197–232.

Google Scholar

Karatzas, I., and S. G. Kou (1996): On the Pricing of Contingent Claims under Constraints, *Ann. Appl. Prob*. **6**, 321–369.

Google Scholar

Karatzas, I., and S. Shreve (1991): *Brownian Motion and Stochastic Calculus*, 2nd ed. New York : Springer.

Google Scholar

Lamberton, D., and B. Lapeyre (1996): *Introduction to Stochastic Calculus Applied to Finance*. Boca Raton , FL : Chapman & Hall.

Google Scholar

Leland, H. E. (1985): Option Pricing and Replication with Transactions Costs, *Journal of Finance* **40**, 1283–1301.

Web of Science® | Google Scholar

Moore, K. S., and V. R. Young (2002): Pricing Equity-Linked Pure Endowments via the Principle of Equivalent Utility. Working paper, Department of Mathematics, University of Michigan .

Google Scholar

Musiela, M., and T. Zariphopoulou (2002): Indifference Prices and Related Measures. Working paper, University of Texas at Austin .

Google Scholar

Nielsen, J. A., and K. Sandmann (1995): Equity-Linked Life Insurance: A Model with Stochastic Interest Rates, *Insurance: Math. & Econ*. **16**, 225–253.

Web of Science® Google Scholar

Persson, S.-A. (1993): Valuation of a Multistate Life Insurance Contract with Random Benefits, *Scand. J. Mgmt*. **9**, 573–586.

Google Scholar

Pratt, J. W. (1964): Risk Aversion in the Small and in the Large, *Econometrica* **32**, 122–136.

Web of Science® Google Scholar

Renault, E., and N. Touzi (1990): Option Hedging and Implied Volatilities in a Stochastic Volatility Model, *Mathematical Finance* **6**, 279–302.

Google Scholar

Rouge, R., and N. El Karoui (2000): Pricing via Utility Theory and Entropy, *Math. Finance* **10**(2), 259–276.

Web of Science® Google Scholar

Schmock, U. (1999): Estimating the Value of the WINCAT Coupons of the Winterthur Insurance Convertible Bond, *ASTIN Bull*. **29**(1), 101–163.

Google Scholar

Tiong, S. (2000): Valuing Equity-Indexed Annuities, with discussion, *N. Am. Actuarial J*. **4**, 149–170.

Google Scholar

Wang, S. (1995): Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms, *Insurance: Math. & Econ*. **17**, 43–54.

CAS Web of Science® Google Scholar

Wilmott, P., S. Howison, and J. Dewynne (1995): *The Mathematics of Financial Derivatives: A Student Introduction*. Cambridge : Cambridge University Press.

Google Scholar

Young, V. R. (2003): Equity-Indexed Life Insurance: Pricing and Reserving Using the Principle of Equivalent Utility, *N. Am. Actuarial J*. **7**(1), 68–86.

Google Scholar

Young, V. R., and T. Zariphopoulou (2002a): Pricing Dynamic Insurance Risks Using the Principle of Equivalent Utility, *Scand. Actuarial J.* **2002**(4), 246–279.

Google Scholar

Young, V. R., and T. Zariphopoulou (2002b): Pricing Insurance via Stochastic Control: Optimal Consumption and Terminal Wealth. Working paper, School of Business, University of Wisconsin-Madison .

Google Scholar

Citing Literature

Subscription Agents

Advertisers & Corporate Partners

CONNECT WITH WILEY

The Wiley Network

Wiley Press Room

WILEY