



Mathematical Finance / Volume 16, Issue 4 / p. 647-671

## ASSET ALLOCATION AND ANNUITY-PURCHASE STRATEGIES TO MINIMIZE THE PROBABILITY OF FINANCIAL RUIN

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First published: 01 September 2006

<https://doi.org/10.1111/j.1467-9965.2006.00288.x>

Citations: 75

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We thank an anonymous referee for many helpful comments. We thank Erhan Bayraktar, David Promislow, and Thomas Salisbury for fruitful discussions of this work, and we thank the participants of the 2004 IFID Conference on Risk and Mortality for their feedback.

*Manuscript received June 2004; final revision received March 2005.*

### Abstract

In this paper, we derive the optimal investment and annuitization strategies for a retiree whose objective is to minimize the probability of lifetime ruin, namely the probability that a fixed consumption strategy will lead to zero wealth while the individual is still alive. Recent papers in the insurance economics literature have examined utility-maximizing annuitization strategies. Others in the probability, finance, and risk management literature have derived shortfall-minimizing investment and hedging strategies given a limited amount of initial capital. This paper brings the two strands of research together. Our model pre-supposes a retiree who does not currently have sufficient wealth to purchase a life annuity that will yield her exogenously desired fixed consumption level. She seeks the asset allocation and annuitization strategy that will minimize the probability of lifetime ruin. We demonstrate that because of the binary nature of the investor's goal, she will not annuitize any of her wealth until she can fully cover her desired consumption with a life annuity. We derive a variational inequality that governs the ruin probability and the optimal strategies, and we demonstrate that the problem can be recast as a related optimal stopping problem which yields a free-boundary problem that is more tractable. We numerically calculate the ruin probability and optimal strategies and examine how they change as we vary the mortality assumption and parameters of the financial model. Moreover, for the special case of exponential future lifetime, we solve the (dual) problem explicitly. As a byproduct of our calculations, we are able to quantify the reduction in lifetime ruin probability that comes from being able to manage the investment portfolio dynamically and purchase annuities.

Bowers, N. L., H. U. Gerber, J. C. Hickman, D. A. Jones, and C. J. Nesbitt (1997): *Actuarial Mathematics*, second edition. IL : Schaumburg , Society of Actuaries.

[Google Scholar](#)

---

Brown, J. R. (2001): Private Pensions, Mortality Risk, and the Decision to Annuitize, *J. Public Econ.* **82**(1), 29–62.

[Google Scholar](#)

---

Browne, S. (1995): Optimal Investment Policies for a Firm with a Random Risk Process: Exponential Utility and Minimizing the Probability of Ruin, *Math. Oper. Res.* **20**(4), 937–958.

[Web of Science®](#) | [Google Scholar](#)

---

Browne, S. (1999a): Beating a Moving Target: Optimal Portfolio Strategies for Outperforming a Stochastic Benchmark, *Finance Stoch.* **3**, 275–294.

[Web of Science®](#) | [Google Scholar](#)

---

Browne, S. (1999b): The Risk and Rewards of Minimizing Shortfall Probability, *J. Portfolio Manag.* **25**(4), 76–85.

[Web of Science®](#) | [Google Scholar](#)

---

Browne, S. (1999c): Reaching Goals by a Deadline: Digital Options and Continuous-Time Active Portfolio Management, *Adv. Appl. Probab.* **31**, 551–577.

[Web of Science®](#) | [Google Scholar](#)

---

Brugiavini, A. (1993), Uncertainty Resolution and the Timing of Annuity Purchases, *J. Public Econ.* **50**, 31–62.

[Web of Science®](#) | [Google Scholar](#)

---

Copeland, C. (2002), An Analysis of the Retirement and Pension Plan Coverage Topical Module of SIPP, Issue Brief of the Employee Benefit Research Institute , <http://www.ebri.org>, May 2002.

[Google Scholar](#)

---

Cummins, H. J. (2004): Bill Rewards Buyers of Life Annuities; Plan Would Give Retirees Half of the Income Tax-Free, *Star Tribune*, Minneapolis , Minnesota , July 29, 2004.

[Google Scholar](#)

---

Davidoff, T., J. Brown, and P. Diamond (2003): Annuities and Individual Welfare, M.I.T. Department of Economics Working Paper Series , Working Paper 03-15.

[Google Scholar](#)

---

Davis, M. H. A., and A. R. Norman (1990): Portfolio Selection with Transaction Costs, *Math. Oper. Res.* **15**, 676–713.

[Web of Science®](#) | [Google Scholar](#)

---

Dixit, A. K., and R. S. Pindyck (1994): *Investment under Uncertainty*. Princeton, NJ : Princeton University Press.

[Google Scholar](#)

---

Duffie, D., W. Fleming, H. M. Soner, and T. Zariphopoulou (1997): Hedging in Incomplete Markets with HARA Utility, *J. Econ. Dyn. Contr.* **21**, 753–782.

[Web of Science®](#) | [Google Scholar](#)

---

Feldstein, M., and E. Rangelova (2001): Individual Risk in an Investment-Based Social Security System, *Am. Econ. Rev.* **91**(4), 1116–1125.

[Web of Science®](#) | [Google Scholar](#)

---

Friedman, A., and W. Shen (2002): A Variational Inequality Approach to Financial Valuation of Retirement Benefits Based on Salary, *Finance Stoch.* **6**(3), 273–302.

[Web of Science®](#) | [Google Scholar](#)

---

Gerber, H. U. (1979): *An Introduction to Mathematical Risk Theory*, S.S. Heubner Foundation Monograph Series, 8, University of Pennsylvania, Wharton School, Philadelphia .

[Google Scholar](#)

---

Harrison, J. M., and M. I. Taksar (1983): Instantaneous Control of Brownian Motion, *Math. Oper. Res.* **8**(3), 439–453.

[Web of Science®](#) | [Google Scholar](#)

---

Huang, H., M. A. Milevsky, and J. Wang (2004): Ruined Moments in Your Life: How Good Are the Approximations? *Insurance: Math. Econ.* **34**(3), 421–447.

[Web of Science®](#) | [Google Scholar](#)

---

Kapur, S., and M. Orszag (1999): A Portfolio Approach to Investment and Annuitization during Retirement, Working Paper, Birkbeck College, University of London .

[Google Scholar](#)

---

Karatzas, I., and S. Shreve (1998): *Methods of Mathematical Finance*. New York : Springer-Verlag.

[Google Scholar](#)

---

Koo, H. K. (1998): Consumption and Portfolio Selection with Labor Income: A Continuous Time Approach, *Math. Fin.* **8**, 49–65.

[Web of Science®](#) | [Google Scholar](#)

---

Milevsky, M. A., and V. R. Young (2003): Annuitization and Asset Allocation, Working Paper, Schulich School of Business, York University .

[Google Scholar](#)

---

Milevsky, M. A., and C. Robinson (2000): Self-Annuitization and Ruin in Retirement, *N. Am. Actuarial J.* **4**(4), 112–129.

[Google Scholar](#)

---

Moore, K. S., and V. R. Young (2006): Optimal and Simple, Nearly Optimal Rules for Minimizing the Probability of Ruin in Retirement, Submitted for publication.

[Google Scholar](#)

---

Neuberger, A. (2002): Optimal Annuitization Strategies, Working Paper, London Business School .

[Google Scholar](#)

---

Øksendal, B. (1998): *Stochastic Differential Equations: An Introduction with Applications*, 5th edition. Berlin : Springer-Verlag.

[Google Scholar](#)

---

Parikh, A. N. (2003): The Evolving U.S. Retirement System, *The Actuary*, March: 2–6.

[Google Scholar](#)

---

Poterba, J. M. (1997): The History of Annuities in the United States, NBER Working Paper 6004.

[Google Scholar](#)

---

Richard, S. (1975): Optimal Consumption, Portfolio and Life Insurance Rules for an Uncertain Lived Individual in a Continuous Time Model, *J. Financ. Econ.* **2**, 187–203.

[Google Scholar](#)

---

Roy, A. D. (1952): Safety First and the Holding of Assets, *Econometrica* **20**, 431–439.

[Web of Science®](#) | [Google Scholar](#)

---

Shreve, S. E., and H. M. Soner (1994): Optimal Investment and Consumption with Transaction Costs, *Ann. Appl. Probab.* 4(3), 206–236.

[Google Scholar](#)

---

SOCIETY OF ACTUARIES (2004): *Risks of Retirement—Key Findings and Issues*, <http://www.soa.org>.

[Google Scholar](#)

---

VanDerhei, J., and C. Copeland (2003): Can America Afford Tomorrow's Retirees: Results from the EBRI-ERF Retirement Security Projection Model, Issue Brief of the Employee Benefit Research Institute , <http://www.ebri.org>, November 2003.

[Google Scholar](#)

---

Wilmott, P., J. Dewynne, and S. Howison (2000): *Option Pricing: Mathematical Models and Computation*. Oxford : Oxford Financial Press.

[Google Scholar](#)

---

Yaari, M. E. (1965): Uncertain Lifetime, Life Insurance and the Theory of the Consumer, *Rev. Econ. Stud.* 32, 137–150.

[Web of Science®](#) | [Google Scholar](#)

---

Young, V. R. (2004): Optimal Investment Strategy to Minimize the Probability of Lifetime Ruin, *N. Am. Actuarial J.* 8(4), 106–126.

[Google Scholar](#)

---

Zariphopoulou, T. (1992): Investment/Consumption Models with Transaction Costs and Markov-Chain Parameters, *SIAM J. Control Optim.* 30: 613–636.

[Web of Science®](#) | [Google Scholar](#)

---

Zariphopoulou, T. (1999): Transaction Costs in Portfolio Management and Derivative Pricing, in *Introduction to Mathematical Finance*, D. C. Heath and G. Swindle, eds. Providence , RI : American Mathematical Society. *Proceedings of Symposia in Applied Mathematics*, 57, 101–163.

[Google Scholar](#)

---

Zariphopoulou, T. (2001): Stochastic Control Methods in Asset Pricing, in *Handbook of Stochastic Analysis and Applications*, D. Kannan and V. Lakshmikantham, eds. New York : Marcel Dekker.

[Google Scholar](#)

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