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ASSET ALLOCATION AND ANNUITY-PURCHASE STRATEGIES TO MINIMIZE THE PROBABILITY OF FINANCIAL RUIN

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Abstract

In this paper, we derive the optimal investment and annuitization strategies for a retiree whose objective is to minimize the probability of lifetime ruin, namely the probability that a fixed consumption strategy will lead to zero wealth while the individual is still alive. Recent papers in the insurance economics literature have examined utility-maximizing annuitization strategies. Others in the probability, finance, and risk management literature have derived shortfall-minimizing investment and hedging strategies given a limited amount of initial capital. This paper brings the two strands of research together. Our model pre-supposes a retiree who does not currently have sufficient wealth to purchase a life annuity that will yield her exogenously desired fixed consumption level. She seeks the asset allocation and annuitization strategy that will minimize the probability of lifetime ruin. We demonstrate that because of the binary nature of the investor's goal, she will not annuitize any of her wealth until she can fully cover her desired consumption with a life annuity. We derive a variational inequality that governs the ruin probability and the optimal strategies, and we demonstrate that the problem can be recast as a related optimal stopping problem which yields a free-boundary problem that is more tractable. We numerically calculate the ruin probability and optimal strategies and examine how they change as we vary the mortality assumption and parameters of the financial model. Moreover, for the special case of exponential future lifetime, we solve the (dual) problem explicitly. As a byproduct of our calculations, we are able to quantify the reduction in lifetime ruin probability that comes from being able to manage the investment portfolio dynamically and purchase annuities.

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