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PORTFOLIO CHOICE VIA QUANTILES

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Abstract

A portfolio choice model in continuous time is formulated for both complete and incomplete markets, where the quantile function of the terminal cash flow, instead of the cash flow itself, is taken as the decision variable. This formulation covers a wide body of existing and new models with law-invariant preference measures, including expected utility maximization, mean–variance, goal reaching, Yaari's dual model, Lopes' SP/A model, behavioral model under prospect theory, as well as those explicitly involving VaR and CVaR in objectives and/or constraints. A solution scheme to this quantile model is proposed, and then demonstrated by solving analytically the goal-reaching model and Yaari's dual model. A general property derived for the quantile model is that the optimal terminal payment is anticomontonic with the pricing kernel (or with the minimal pricing kernel in the case of an incomplete market if the investment opportunity set is deterministic). As a consequence, the mutual fund theorem still holds in a market where rational and irrational agents co-exist.

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