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PORTFOLIO CHOICE VIA QUANTILES

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Abstract

A portfolio choice model in continuous time is formulated for both complete and incomplete markets, where the quantile function of the terminal cash flow, instead of the cash flow itself, is taken as the decision variable. This formulation covers a wide body of existing and new models with law-invariant preference measures, including expected utility maximization, mean–variance, goal reaching, Yaari's dual model, Lopes' SP/A model, behavioral model under prospect theory, as well as those explicitly involving VaR and CVaR in objectives and/or constraints. A solution scheme to this quantile model is proposed, and then demonstrated by solving analytically the goal-reaching model and Yaari's dual model. A general property derived for the quantile model is that the optimal terminal payment is anticomonotonic with the pricing kernel (or with the minimal pricing kernel in the case of an incomplete market if the investment opportunity set is deterministic). As a consequence, the mutual fund theorem still holds in a market where rational and irrational agents co-exist.

REFERENCES

Berkelaar, A., R. Kouwenberg, and T. Post (2004): Optimal Portfolio Choice under Loss Aversion, *Rev. Econ. Stat*. **86**(4), 973–987.

Web of Science® | Google Scholar

Browne, S. (1999): Reaching Goals by a Deadline: Digital Options and Continuous-time Active Portfolio Management, *Adv. Appl. Probab*. **31**(2), 551–577.

Web of Science® | Google Scholar

Browne, S. (2000): Risk-constrained Dynamic Active Portfolio Management, *Manage. Sci*. **46**(9), 1188–1199.

Web of Science® | Google Scholar

Carlier, G., and R.-A. Dana (2006): Law Invariant Concave Utility Functions and Optimization Problems with Monotonicity and Comonotonicity Constraints, *Stat. Decis*. **24**, 127–152.

Google Scholar

Cherny, A., and D. B. Madan (2009): New Measures for Performance Evaluation, *Rev. Financ. Stud.* **22**(7), 2371–2406.

Google Scholar

Cvitanić, J., and I. Karatzas (1992): Convex Duality in Constrained Portfolio Optimization, *Ann. Appl. Probab*. **2**(4), 767–818.

Google Scholar

Das, S. R., H. Markowitz, J. Scheid, and M. Statman (2010): Portfolio Optimization with Mental Accounts, *J. Financ. Quantit. Anal*. **45**(2), 311–344.

Web of Science® | Google Scholar

Dhaene, J., S. Vanduffel, M. J. Goovaerts, R. Kaas, and D. Vyncke (2005): Commonotonic Approximations for Optimal Portfolio Selection Problems, *J. Risk Insur*. **72**(2), 253–300.

Web of Science® | Google Scholar

Dhaene, J., S. Vanduffel, Q. Tang, M. J. Goovaerts, R. Kaas, and D. Vyncke (2006): Risk Measure and Comonotonicity: A Review, *Stochas. Models* **22**(4), 573–606.

Google Scholar

Dybvig, P. H. (1988): Distributional Analysis of Portfolio Choice, *J. Business* **61**(3), 369–398.

Web of Science® Google Scholar

Föllmer, H., and D. Kramkov (1997): Optional Decompositions under Constraints, *Probab. Theory Relat.Fields* **109**(1), 1–25.

Web of Science® Google Scholar

Föllmer, H., and P. Leukert (1999): Quantile Hedging, *Finance Stochas*. **3**, 251–273.

Google Scholar

Föllmer, H., and A. Schied (2004): *Stochastic Finance: An Introduction in Discrete Time*, 2nd edn. Berlin : Walter de Gruyter.

Google Scholar

Goll, T., and L. Rüschendorf (2001): Minimax and Minimal Distance Martingale Measures and Their Relationship to Portfolio Optimization, *Finance Stochas*. **5**(4), 557–581.

Google Scholar

Grigor'ev, P. V., and Y. S. Kan (2004): Optimal Control of the Investment Portfolio with Respect to the Quantile Criterion, *Autom. Remote Control* **65**(2), 319–336.

Web of Science® Google Scholar

Hamada, M., M. Sherris, and J. Van Der Hoek (2006): Dynamic Portfolio Allocation, the Dual Theory of Choice and Probability Distortion Functions, *ASTIN Bull.* **36**(1), 187–217.

Web of Science® Google Scholar

Harrison, J. M., and D. M. Kreps (1979): Martingales and Arbitrage in Multiperiod Security Markets, *J. Econ. Theory* **20**(3), 381–408.

Web of Science® Google Scholar

Harrison, J. M., and S. R. Pliska (1981): Martingales and Stochastic Integrals in the Theory of Continuous Trading, *Stochas. Processes Appl*. **11**(3), 215–260.

Web of Science® Google Scholar

Harrison, J. M., and S. R. Pliska (1983): A Stochastic Calculus Model of Continuous Trading: Complete Markets, *Stochas. Processes Appl*. **15**(3), 313–316.

Google Scholar

He, X. D., and X. Y. Zhou (2008): SP/A Model in Continuous Time, Working Paper, University of Oxford .

Google Scholar

Jacka, S. D. (1992): A Martingale Representation Result and an Application to Incomplete Financial Markets, *Math. Finance* **2**(4), 239–250.

Google Scholar

Jin, H., and X. Y. Zhou (2008): Behavioral Portfolio Selection in Continuous Time, *Math. Finance* **18**, 385–426. Erratum, to appear in *Math. Finance*, 2010.

Web of Science® | Google Scholar

Kahneman, D., and A. Tversky (1979): Prospect Theory: An Analysis of Decision under Risk, *Econometrica* **47**, 263–291.

Web of Science® | Google Scholar

Karatzas, I., and S. E. Shreve (1998): *Methods of Mathematical Finance*, Springer.

Google Scholar

Kataoka, S., (1963): On a Stochastic Programming Model, *Econometrica* **31**, 181–196.

Web of Science® | Google Scholar

Kramkov, D., and W. Schachermayer (1999): The Asymptotic Elasticity of Utility Functions and Optimal Investment in Incomplete Markets, *Ann. Appl. Probab*. **9**(3), 904–950.

Web of Science® | Google Scholar

Kulldorff, M. (1993): Optimal Control of Favorable Games with a Time Limit, *SIAM J. Contl. Optimizat*. **31**(1), 52–69.

Web of Science® | Google Scholar

Lopes, L. L. (1987): Between Hope and Fear: The Psychology of Risk, *Adv. Exp. Social Psychol*. **20**, 255–295.

Web of Science® | Google Scholar

Lopes, L. L., and G. C. Oden (1999): The Role of Aspiration Level in Risk Choice: A Comparison of Cumulative Prospect Theory and sp/a Theory, *J. Math. Psychol*. **43**(2), 286–313.

CAS | PubMed | Web of Science® | Google Scholar

Machina, M. J. (1982): "Expected Utility" Analysis without the Independence Axiom, *Econometrica* **50**(2), 277–322.

Web of Science® | Google Scholar

Merton, R. C. (1969): Lifetime Portfolio Selection under Uncertainty: The Continuous-time Case, *Rev. Econ. Stat*. **51**(3), 247–257.

Web of Science® | Google Scholar

Merton, R. C. (1971): Optimum Consumption and Portfolio Rules in a Continuous-time Model, *J. Econ. Theory* **3**, 373–413.

Web of Science® | Google Scholar

Pliska, S. R. (1986): A Stochastic Calculus Model of Continuous Trading: Optimal Portfolios, *Math. Operat. Res*. **11**(2), 371–382.

Web of Science® | Google Scholar

Quiggin, J. (1982): A Theory of Anticipated Utility, *J. Econ. Behav. Organizat*. **3**, 323–343.

Web of Science® | Google Scholar

Rothschild, M., and J. E. Stiglitz (1970): Increasing Risk: I. A Definition, *J. Econ. Theory* **2**(3), 225–243.

Web of Science® | Google Scholar

Samuelson, P. A. (1969): Lifetime Portfolio Selection by Dynamic Stochastic Programming, *Rev. Econ. Stat*. **51**(3), 239–246.

Web of Science® | Google Scholar

Schachermayer, W., M. Sirbu, and E. Taflin (2009): In Which Financial Markets Do Mutual Fund Theorems Hold True?, *Finance Stochas*. **13**(1), 49–77.

Web of Science® | Google Scholar

Schied, A. (2004): On the Neyman-Pearson Problem for Law-invariant Risk Measures and Robust Utility Functionals, *Ann. Appl. Probab*. **14**, 1398–1423.

Web of Science® | Google Scholar

Schmeidler, D. (1989): Subjective Probability and Expected Utility without Additivity, *Econometrica* **57**(3), 571–587.

Web of Science® | Google Scholar

Spivak, G., and J. Cvitanić: Maximizing the Probability of a Perfect Hedge, *Ann. Appl. Probab*. **9**(4), 1303–1328.

Google Scholar

Thaler, R. (1980): Towards a Positive Theory of Consumer Choice, *J. Econ. Behav. Organizat*. **1**, 39–60.

Web of Science® | Google Scholar

Tversky, A., and D. Kahneman (1992): Advances in Prospect Theory: Cumulative Representation of Uncertainty, *J. Risk Uncertainty* **5**, 297–323.

Web of Science® | Google Scholar

Yaari, M. E. (1987): The Dual Theory of Choice under Risk, *Econometrica* **55**(1), 95–115.

Web of Science® | Google Scholar

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