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Econometric Analysis of Realized Covariation: High Frequency Based Covariance, Regression, and Correlation in Financial Economics

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Abstract

This paper analyses multivariate high frequency financial data using realized covariation. We provide a new asymptotic distribution theory for standard methods such as regression, correlation analysis, and covariance. It will be based on a fixed interval of time (e.g., a day or week), allowing the number of high frequency returns during this period to go to infinity. Our analysis allows us to study how high frequency correlations, regressions, and covariances change through time. In particular we provide confidence intervals for each of these quantities.

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