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Article

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THE TAX-ADJUSTED YIELD CURVE

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IT HAS BEEN DEMONSTRATED, by Robichek and Niebuhr [8], that tax-induced bias can substantially alter the shape of the yield curve if it is constructed from quotations on bonds selling below par. The apparent before tax yield curve can be upward sloping at the same time that the tax-adjusted yield curve is downward sloping. Thus, the inclusion of tax effects can actually reverse qualitative conclusions concerning the direction in which investors expect interest rates to move under the expectations hypothesis.

Furthermore, simple before tax term structure estimation does not satisfactorily explain the market prices of low coupon bonds selling at a discount, because of the long-term capital gains tax advantage on these securities. Treating these bonds as outliers is unsatisfactory, since they constitute the bulk of observations for some maturities.[1]

The present paper modifies our technique for regression fitting the term structure of interest rates, described in an earlier paper [4], to eliminate this tax-induced bias and reconcile observations on high and low coupon bonds.

I. SECURITY PRICES AND THE DISCOUNT FUNCTION

As in our earlier paper, it is most convenient to begin with the discount function $\delta(m)$. In the present paper, this curve gives the present value of \$1.00 *after tax* repayable after m years.

If we could ignore tax effects, the price of a bond with par value 100, coupon rate c, and terminal maturity date m would be given by

$$p = 100\delta(m) + c\int_0^m \delta(\mu)d\mu.^2 \qquad (1)$$

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1. Fisher [1] partially adjusts for this effect by including coupon terms in his yield-curve regression. However, directly fitting the yield curve is not based on the summation principle of equation (1). Weingartner [9] compensates for coupons through a series of approximations in a way that gives a yield curve similar to our regression-produced yield curve in [4]. However, he does not adjust for tax effects. Williams [10, chapters 10 and 20] gives an algorithm for fitting security prices exactly that uses the summation principle, although he too makes no adjustment for taxes. McCallum [3] and Pye [6] treat the effect of capital gains taxation, though not in a term-structure context.

2. As in [4], we assume for the sake of simplicity that coupons arrive in a continuous stream, instead of in semi-annual installments. We therefore interpret p as the quoted "and interest" price, rather than as the "flat" price at which the security actually changes hands. This convention simplifies the analysis and reduces computation time, but introduces a slight inaccuracy, especially in the maturities where bills interface with short-term notes and bonds. (This problem was called to my attention by Allen Lerman.) This difficulty can be easily corrected by replacing the integrals in the

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Citing Literature

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