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Optimal Stopping With Multiple Priors

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Abstract

We develop a theory of optimal stopping under Knightian uncertainty. A suitable martingale theory for multiple priors is derived that extends the classical dynamic programming or Snell envelope approach to multiple priors. We relate the multiple prior theory to the classical setup via a minimax theorem. In a multiple prior version of the classical model of independent and identically distributed random variables, we discuss several examples from microeconomics, operation research, and finance. For monotone payoffs, the worst-case prior can be identified quite easily with the help of stochastic dominance arguments. For more complex payoff structures like barrier options, model ambiguity leads to stochastic changes in the worst-case beliefs.

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