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THE QUADRATIC APPROXIMATION TO THE YIELD TO MATURITY

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The yield to maturity (YTM) of a bond is a measure of the rate of return on the capital invested in that bond[2], for which financial theorists and practitioners alike have sought more accurate approximate equations[3]. Many have been suggested, as documented by Hawawini and Vora [6], and most of them belong to a one-parameter family which may be written as:

$$AY = \frac{C + (F - P)/n}{\frac{(F + \alpha P)}{+1}} \tag{1}$$

where AY represents the approximate yield, C is the periodic coupon payment, P is the bond's current price, n is the number of periods remaining to maturity, F is the bond's face value, and α is the parameter which generates the family of equations.

The purpose of this paper is to find the constant value of α in Equation 1 which results in the most accurate estimate of the YTM. The search is restricted to constant values of α so that the most accurate equation retains the desirable simplicity of the traditional textbook equation, which uses $\alpha = 1$, e.g., see Brigham [2, p. 118], and Cooley and Roden [3, p.167][4].

The continuing appeal of the traditional equation is due to its simplicity and to the fact that it can be interpreted as the ratio of the average total return to the average price of the bond, see Cooley and Roden [3, p.167]. In fact, this interpretation is valid for any member of the family generated by Equation 1. The key lies in assuming that the bond's price path over time may be approximated by means of a polynomial function. The traditional equation, for example, implicitly assumes that the price of

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[2]The YTM is usually expressed as a nominal annual rate. Since most bonds pay coupons semiannually, to get the effective annual YTM the following transformation is required: $1 + YTM_e = (1 + YTM_n/2)^2$. Thus, the nominal YTM understates the effective YTM. For an extensive discussion of this point see Lindley, Helms, and Haddad [8].

[3]Abel proved that an algebraic solution for the YTM does not exist for $n \geq 5$, see [9]. In practice, though, obtaining an exact equation for the YTM for maturities greater than $n = 2$ may be a formidable task.