



Handbook of Heavy Tailed Distributions in Finance

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Chapter 1 - Heavy Tails in Finance for Independent or Multifractal Price Increments

[Benoit B. Mandelbrot](#)

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Abstract

This chapter has two goals. Section 1 sketches the history of heavy tails in finance through the author's three successive models of the variation of a financial price: mesofractal, unifractal and multifractal. The heavy tails occur, respectively, in the marginal distribution only (Mandelbrot, 1963), in the dependence only (Mandelbrot, 1965), or in both (Mandelbrot, 1997). These models increase in the scope of the “principle of scaling invariance”, which the author has used since 1957.

The mesofractal model is founded on the stable processes that date to Cauchy and Lévy. The unifractal model uses the fractional Brownian motions introduced by the author. By now, both are well-understood.

To the contrary, one of the key features of the multifractals (Mandelbrot, 1974a, b) remains little known. Using the author's recent work, introduced for the first time in this chapter, the exposition can be unusually brief and mathematically elementary, yet covering all the key features of multifractality. It is restricted to very special but powerful cases: (a) the Bernoulli binomial measure, which is classical but presented in a little-known fashion, and (b) a new two-valued “canonical” measure. The latter generalizes Bernoulli and provides an especially short path to negative dimensions, divergent moments, and divergent (i.e., long range) dependence. All those features are now obtained as separately tunable aspects of the same set of simple construction rules.

My work in finance is well-documented in easily accessible sources, many of them reproduced in Mandelbrot (1997 and also in 2001a, b, c, d). That work having expanded and been commented upon by many authors, a survey of the literature is desirable, but this is a task I cannot undertake now. However, it was a pleasure to yield to the entreaties of this Handbook's editors by a text in which a new technical contribution is preceded by an introductory sketch followed by a simple new presentation of an old feature that used to be dismissed as “technical”, but now moves to center stage.

The history of heavy tails in finance began in 1963. While acknowledging that the successive increments of a financial price are interdependent, I assumed independence as a first approximation and combined it with the principle of scaling invariance. This led to (Lévy) stable distributions for the price changes. The tails are very heavy, in fact, power-law distributed with an exponent $\alpha < 2$.

The multifractal model advanced in Mandelbrot (1997) extends scale invariance to allow for dependence. Readily controllable parameters generate tails that are as heavy as desired and can be made to follow a power-law with an exponent in the range $1 < \alpha < \infty$. This last result, an essential one, involves a property of multifractals that was described in Mandelbrot (1974a, b) but remains little known among users. The goal of the example described after the introduction is to illustrate this property in a very simple form.

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