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Financial valuation of guaranteed minimum withdrawal benefits

Moshe A. Milevsky [a], Thomas S. Salisbury [b]

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Abstract

Financial valuation OF GMWBs: We develop a variety of methods for assessing the cost and value of a very popular 'rider' available to North American investors on variable annuity (VA) policies called a Guaranteed Minimum Withdrawal Benefit (GMWB). The GMWB promises to return the entire initial investment, albeit spread over an extended period of time, regardless of subsequent market performance. First, we take a *static* approach that assumes individuals behave passively and holds the product to maturity. We show how the product can be decomposed into a Quanto Asian Put plus a generic term-certain annuity. At the other extreme of consumer behavior, the *dynamic* approach leads to an optimal stopping problem akin to pricing an American put option, albeit complicated by the non-traditional payment structure. Our main result is that the No Arbitrage hedging cost of a GMWB ranges from 73 to 160 basis points of assets. In contrast, most products in the market only charge 30–45 basis points. Although we suggest a number of behavioral reasons for the apparent under-pricing of this feature in a typically overpriced VA market, we conclude by arguing that current pricing is not sustainable and that GMWB fees will eventually have to increase or product design will have to change in order to avoid blatant arbitrage opportunities.

Section snippets

Introduction and motivation

"*They have stumbled onto a 'killer app' for the financial needs of today's boomers, It's called a GMWB. The deal is that for a half-percentage point a year, you can invest with a guarantee that your entire principal will be returned to you, provided that the principal is not withdrawn at a rate greater than 7% annually* . . ." Washington Post, May 23, 2004…

"The risks of variable annuities have come home to roost for insurers ... To make matters worse, rating agencies, accountants and regulators never…

…

Static modeling framework

Let W_t denote the market value of the underlying VA at any future time $t \geq 0$, with an arbitrary (but innocuous) assumption that $w_0 = 100$ dollars. The most typical GMWB structure is that the policyholder is guaranteed to be able to withdraw *at least* $G = gw_0 = 7$ dollars per annum. The guarantee remains in effect until the entire $ 100 has been disbursed which, at minimum, is a period of $100/7 = 14.28$ years. Thus, even in the extreme scenario where the initial $w_0 = 100$ collapses to a zero value 1 day after…

Static analysis

In this section we illustrate how to bifurcate the product into a collection of strip-bonds (or a term-certain annuity) and a Quanto Asian Put (QAP) option. Note that $g = G/w_0$ and by definition $T = 1/g$ (since the product terminates or matures when all the funds have been returned) and so we have that:

$W_T = w_0\, \mathrm{e}^{(\mu-\alpha-(1/2)\sigma^2)T+\sigma B_T} \max \times$ The payoff of the *QAP option* is:
$\left[0, \left(1 - \frac{1}{T}\int_0^T \mathrm{e}^{-(\mu-\alpha-(1/2)\sigma^2)s-\sigma B_s}\, \mathrm{d}s\right)\right].$

$\textbf{Option Payoff} := W_T$, since the holder of the variable annuity policy is guaranteed to receive any remaining…

Dynamic model and hedging

In this section we employ *American option* pricing techniques to obtain a dynamic model of the GMWB, assuming policyholders are fully rational and lapse (i.e. withdraw more or less from) the product when it works to their economic advantage. As we argued in the introduction, the true 'cost' of the embedded guarantee lies somewhere between the *static* embedded option cost and the *dynamic* hedging cost.

It is important to note that once we include strategic lapsation as an option for the…

Conclusion

In this paper we develop two approaches for analyzing a novel type of derivative security that has become quite popular in North America—called a Guaranteed Minimum Withdrawal Benefit (GMWB)—which is an insurance rider offered on Variable Annuity (VA) policies. VA policies are a close cousin to mutual funds, but offer additional performance-based guarantees. First, we take a *static* approach that assumes individual investors behave passively in utilizing the embedded guarantee. In this case, we…

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