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# Reduced-form valuation of callable corporate bonds: Theory and evidence ☆

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## Abstract

We develop a reduced-form approach for valuing callable corporate bonds by characterizing the call probability via an intensity process. Asymmetric information and market frictions justify the existence of a call-arrival intensity from the market's perspective. Our approach both extends the reduced-form model of Duffie and Singleton (1999) for defaultable bonds to callable bonds and captures some important differences between call and default decisions. A comprehensive empirical analysis of callable bonds using both our model and the more traditional American option approach for valuing callable bonds shows that the reduced-form model fits callable bond prices well and that it outperforms the traditional approach both in- and out-of-sample.

## Introduction

One of the most exciting developments in the fixed-income literature in the last decade is the reduced-form model for corporate bonds pioneered by Jarrow and Turnbull (1995) and Duffie and Singleton (1999) (DS hereafter). By modeling the default probability as an intensity process, this approach greatly simplifies the valuation of corporate bonds and makes large-scale empirical analysis of corporate bonds possible. In addition to corporate bonds, empirical studies have applied this approach to a wide variety of securities with default risk, such as sovereign bonds, interest rate swaps, and credit derivatives; Dai and Singleton (2003) provide a survey of the empirical literature on dynamic term structure models for default-free and defaultable bonds. A striking feature of this literature is that most empirical studies have focused only on non-callable bonds, even though as pointed out by DS, "the majority of dollar-denominated corporate bonds are callable."[1] The dominant reason for this omission is that callable bonds are more difficult to price and, at present, there exist no accurate and tractable valuation methods suitable for a large-scale empirical analysis of callable corporate bonds.

Indeed, the traditional structural approach for valuing callable corporate bonds typically assumes a stochastic process for firm value and determines the optimal call policy by minimizing the present value of its liabilities. Determining this optimal call policy in the presence of default risk and possibly other callable bonds is a challenging task. It requires knowledge of both the firm's value process and the firm's dynamic liability structure, which are typically unobservable to market participants. Furthermore, it is difficult to fully incorporate the impact of market frictions, which can significantly change the optimal call policy.[2] And even given all of these elements, valuation itself is an enormous computational exercise (see Jones, Mason, and Rosenfeld, 1984; Berndt, 2004). Therefore, while the structural approach provides useful insights on the valuation of callable bonds (see Acharya and Carpenter, 2002), it is very difficult to implement in practice.

Given the limitations of the structural approach, DS propose to value callable corporate bonds as American options written on otherwise identical non-callable bonds valued using the reduced-form approach. Although this does not require information on the firm's assets and capital structure, it assumes that the firm calls the bond to minimize the market value of the particular bond under analysis. This assumption ignores the bond's impact on the firm's remaining liabilities, and hence shareholder's equity might not be maximized. The optimal call policy determination also ignores market frictions. Finally, the American feature is computationally difficult because valuing American interest rate options is numerically challenging. As a result, empirical implementations of this approach tend to be of limited scale. For example, in one of the few applications of this approach, Berndt (2004) considers only one firm, Occidental Petroleum.

The objectives of our paper are (i) to develop a valuation method for callable corporate bonds that is as tractable as the reduced-form model for credit risk yet still captures the relevant differences between call and default decisions, and (ii) to apply the new method in a comprehensive empirical pricing analysis of callable corporate bonds. Thus, the contributions of our paper are both methodological and empirical.

Methodologically, we extend the reduced-form model for non-callable bonds to callable bonds by modeling the firm's call policy, from the market's perspective, via a stochastic intensity process.[3] The existence of a call intensity process can be justified on two grounds. First, analogous to the justification of reduced-form credit risk models given by Duffie and Lando (2001) and Cetin, Jarrow, Protter, and Yildirim (2004), since the firm's true value is not observable, the optimal call policy can indeed be viewed as an inaccessible stopping time from the market's perspective (see Li, Liu, and Wu, 2003). Second, market frictions such as transaction costs, illiquidity, and other idiosyncratic features can cause premature or delayed call exercise (e.g., Vu, 1986, Longstaff, 1992). As such, analogous to mortgage bond prepayments, transaction costs can also result in a call-arrival intensity process (see Stanton, 1995). The existence of a call intensity process allows a reduced-form representation for callable bond valuation.

However, our model is not just a simple extension of the reduced-form model for non-callable bonds, because of important differences between the call and default decisions. When a firm defaults on one of its bonds due to bond covenants, it defaults on all its bonds. But when a firm decides to call a bond, standard economic theory predicts that it might not call all outstanding bonds. Only bonds whose market value exceeds the call price should be exercised. Consequently, high-coupon bonds are more likely to be called than low-coupon bonds. Of course, all callable bonds are more likely to be called when interest rates are low. Moreover, at least in theory, there should be a nonlinear relation between call spreads and interest rates due to the nonlinearity embedded in the call option's payoff. We introduce a parsimonious model for the call spread that (i) explicitly captures the call spread's dependence on the coupon rate and its nonlinear dependence on interest rates, and (ii) provides a closed-form approximation for callable bond prices based on a new method developed by Kimmel (2008). In our model, callable corporate bond yields impound the joint effects of default and the embedded call provision.

Our new reduced-form model greatly simplifies callable bond pricing and has advantages over existing methods. Unlike the structural approach, our approach circumvents the need to determine the firm's optimal call policy and thus does not require information on firm value or capital structure. Unlike the American valuation procedure in DS, our approach allows more complex call exercise policies. These more complex exercise policies are consistent with those observed in practice. Furthermore, our approach is less computationally challenging than that of the structural models or the DS procedure. Indeed, in our reduced-form model, the valuation of callable bonds is computationally equivalent to that of Treasury or non-callable bonds, and closed-form solutions are available for popular model specifications. The flexibility and tractability of our reduced-form approach make it an appealing alternative to existing methods for valuing callable corporate bonds.

We provide one of the first comprehensive empirical analyses of callable bonds under both our reduced-form model and the DS model. Compared to existing studies, our paper considers a relatively large sample of firms that issue both non-callable and callable bonds. We adopt a four-factor affine specification for the default-free term structure, default, and call processes. Our results show that the reduced-form model provides a close fit to callable bond prices and has generally smaller in- and out-of-sample pricing errors than does the American option approach of DS.

While our analysis focuses on the pricing of callable bonds, our reduced-form model has broader applications. For example, our model simplifies the computation of hedge ratios for callable bonds. The multifactor affine model decomposes callable bond yields into a default-free interest rate, a credit spread, and a call spread. Given the estimated dynamics for each component, it is easy to hedge the risk factor exposures of a callable bond. Indeed, the interest rate and credit risks can be hedged using Treasury and non-callable or credit derivatives, respectively. The reasonably good out-of-sample performance of our model suggests that it is possible to partially hedge call risk using another callable bond from the same firm. In contrast, hedging callable bonds in the DS model crucially depends on a correct modeling of the firm's optimal call policy, and the calculation of hedge ratios is computationally difficult.

Our reduced-form model can be extended to price options on callable bonds, just as the reduced-form credit risk model facilitates the pricing of credit derivatives. One interesting example is the pricing of options on mortgage-backed securities (MBS), which have become increasingly popular in the fixed-income market (see Zhou and Subramanian, 2004). Mortgage options differ from standard bond options in that their underlying assets have embedded a prepayment option. Similar to those of callable bonds, the prepayment behaviors of MBS are very complicated and depend on factors that are often outside standard economic models. Given our model's close fit to callable bond prices and the tractability of the affine dynamics, pricing mortgage options using our modeling framework is a natural application. The decomposition of a callable bond's yield into its different components under our model also makes it much easier to hedge a mortgage option's exposure to these risk factors.[4] In contrast, given the difficulties of the existing models in pricing callable bonds and their computational challenges, it seems unlikely that they can be used to price options on callable bonds.

The remainder of the paper is organized as follows. In Section 2, we develop a reduced-form model for callable corporate bonds. In Section 3, we introduce the data underlying our empirical analysis and discuss the econometric methods for estimation. Section 4 contains the empirical results and Section 5 concludes the paper. In the Appendix, we provide a brief discussion on how to apply Kimmel's (2008) method to our model.

## Section snippets

### A reduced-form model for callable corporate bonds

In this section, we develop a reduced-form model for callable corporate bonds by characterizing the call probability as an intensity process. We compare our approach with that of DS, who price callable bonds as American options. We also provide a closed-form formula for callable corporate bonds under an affine model specification.…

### Research design

In this section, we discuss the data and the econometric method for implementing our model and that of DS for callable bonds. Callable bond yields have three components: the risk-free interest rate and the credit and call spreads. We use a two-stage estimation procedure in the determination of these three components. First, we estimate the default-free term structure using yields on Treasury securities. We then use these estimated risk-free processes to price both non-callable bonds (credit…

### The empirical results

This section presents the empirical evidence on the pricing performance of our reduced-form model for callable corporate bonds. After discussing the estimation of the default-free and defaultable term structures, we focus on fitting the callable bond prices. We also compare our model with that of DS for several firms with enough information for both an in- and out-of-sample analysis. We then assess the importance of a liquidity premium in callable bond spreads. Finally, we provide some…

### Conclusion

We develop a reduced-form approach for valuing callable corporate bonds by characterizing the call probability via an intensity process. Asymmetric information and market frictions justify the existence of a call-arrival intensity from the market's perspective. Our approach extends the reduced-form model of Duffie and Singleton (1999) for defaultable bonds to callable bonds in a manner consistent with differences between call and default decisions. We also provide one of the first comprehensive …

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