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Chaos in economics and finance ☆

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Abstract

This paper focuses on the use of dynamical chaotic systems in economics and finance. In these fields, researchers employ different methods from those taken by mathematicians and physicists. We discuss this point. Then, we present statistical tools and problems which are innovative and can be useful in practice to detect the existence of chaotic behavior inside real data sets.

Introduction

Chaotic systems are complex systems which belong to the class of deterministic dynamical systems. They are detected and used in a lot of fields for control or forecasting. Deterministic chaos has been rigorously and extensively studied by mathematicians and other scientists. It is almost impossible to give a precise mathematical definition of deterministic chaos that encapsulates everything in the diverse literature. In this paper we are not interested by complete mathematical rigour, and avoid technical details. Our viewpoint is to understand how the notion of chaotic systems is viewed, analyzed and used in different fields.

As a first insight, we know that these systems are characterized by strong non-linearity, permitting to take into account non-periodic fluctuations, mixing cycles and switches inside data sets. They are characterized by an invariant distribution function and their orbits evolve inside an attractor in which it is possible to do forecasts. Working inside an attractor also permits to control the system and to be able to avoid explosions and strong volatility, which is an interesting task for applications.

Historically, mathematicians were the first to be interested by this theory. Since 1800, work concerning deterministic dynamical systems which generate random behaviors has been investigated, and Poincare's works (Poincaré, 1908) enable those studies to foster a growing interest and to be developed in a lot of

Typesetting math: 100% ogy, people who study population evolution have used chaotic deterministic systems since the 1970s, May (1976). In physics, it is a long tradition for researchers, working mainly with an empirical approach, to use these models. The works of Ruelle and Takens (1971) proposed a new approach to non-linear modelling with few parameters and also gave the opportunity to extend the research in this field in this community. In economics, people working on stability and instability have flirted with bifurcation theory since the 1980s. This approach is really new and unexpected coming from this community. Indeed, working with chaotic systems is in opposition to most of the different concepts developed by macro-economists: we cite for instance, the neo-classical theory of Lucas, Sargent, Prescott, etc., the 'rational' theory which uses mainly linear concepts or, Keynes' theory which is not concerned with complex systems, Medio (1992). Between the years 1986 and 1998, a lot of studies, following the idea of Grandmont (1988), guided a part of the research in economics towards chaotic systems. In particular, it explained randomness endogenously. New econometric models were developed and time irreversibility was introduced. In finance the interest in chaos theory is more recent and sparse. The craze for this theory from financial people began around the nineties. People expected to get robust forecasts using chaos. Finally, chaos has been recently developed as an area of increasing interest for statisticians.

Why are some statisticians so interested by the vast potential which can be gained studying deterministic chaos? One reason is that deterministic dynamical systems can generate chaos, that is highly erratic behavior reminiscent of realizations of random process. Now, in the study of deterministic dynamical systems, environmental noise tends to be suppressed or, at most, plays a secondary role. In statistics, randomness is generated through a stochastic process and we speak about stochastic dynamical systems. What is the link between the deterministic characteristics of a chaotic system free of noise and the property of stochasticity? We can illustrate this fact using the logistic map, largely studied in the literature, May (1976). Consider the following map defined by$X_t = 4X_{t-1}\left(1 - X_{t-1}\right), \quad t = 1,2,\ldots$The solution of the logistic map, given a starting value, is also called a trajectory. If the starting value is between 0 and 1, then all the iterates of the logistic map will remain in the interval [0,1]. The natural measure whose density is $\phi(x) = 1/\left(\pi\sqrt{x(1-x)}\right)$, $0 \leq x \leq 1$, and zero elsewhere, reflects the fact that each point inside [0,1] will be visited arbitrarily closely and infinitely often. This natural measure is associated with a typical trajectory of the previous logistic map and describes the 'frequency' of any point to be visited by such a trajectory and it can be linked with the marginal distribution of the logistic map. Thus, we may introduce stochasticity into the logistic dynamical system by specifying the initial condition according to some probability distribution. Then $X_t, t = 1,2,\ldots$, defined by (1), becomes a random sequence.

It is this link between the notion of chaos and stochastic environment which has been investigated by some statisticians, permitting to offer in real data analysis the possibility to extracting 'chaotic' signals from noisy data sets.

Now, on the other hand, we can remark that, concerning the use of deterministic chaotic systems, each community may have different strategies. They do not use the same models, nor the same information set. Most mathematicians work with analytical expressions and characterize their models under specific assumptions to decide if they can exhibit specific chaotic behaviors, characterized by specific properties, following varied definitions of 'chaos'. The economists generally use analytical systems corresponding to a specific modelling problem. These systems depend on few parameters and one purpose is to detect the range of these parameters in which they can lead to stable or unstable behaviors. Bifurcation theory is often a basis of their studies. Generally economists do not follow the roads of physicists. Indeed, physicists are interested by questions relative to universal laws, and in economics the trend is to understand and document differences. In finance, practitioners do not use analytical systems and want to use chaos theory to robustify their forecasts: most of the work is empirical. In statistics, work concerns estimation theory and tries to prove robustness of estimates of the Lyapunov exponents or the embedding dimension, for instance. They are also interested in re-building orbits and forecasting on the attractors.

Section snippets

What kind of chaos for which models?

Let $\underline{X}_t$ be a random vector characterized by the following equation $\underline{X}_t = f(\underline{X}_{t-1})$,and $\underline{X}_0$ an initial condition. The sequence $(\underline{X}_t)_t$ corresponds to a dynamical deterministic system and we assume that it is characterized by an invariant measure. This system is defined on a metric space $A \in R^d$, $d \in N$ and *f* is a non-linear function: $A \to A$. As a working definition for our purpose, we will say that such a system has chaotic behavior if it is non-linear, if it is characterized by the existence of an attractor…

Statistical tools for chaos theory

In this section, we specify the statistical tools which support the attempts of practitioners in economics and finance when no model is known *a priori*. We do not enter into details, and refer to basic works. Estimation theory is important and also the predictive approach, thus we discuss these two facts.

As we already said, the observation of one trajectory $X_1, \ldots, X_n$ cannot detect the existence of an attractor characterizing the data set $(X_t)_t$. The embedding of the data set is then necessary.

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Open problems

This paper has discussed the approaches followed by some economists and financial researchers when they apply chaos theory to real data sets. The main difference between physicists and economics is due to the fact that in economy, we only have a unique trajectory (and not the possibility to repeat the experience as in physics) and also to presence of measurement noise in the time series.

The presence of noise in real data sets is a brake for the use of chaos theory in practice. Thus, robust…

Dominique Guégan is currently full Professor at the University Paris 1 Panthéon – Sorbonne. She is an expert in Non linear time series, Dynamical systems, parametric and non-parametric statistical techniques. She has already written seven books and participated to more than 10 collective books. She is mainly interested to apply the methodologies she developed in several fields like Finance, Economics, Physics, Environment and Medicine. She has published more than 60 articles in several reviews…

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