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Bootstrapping Parametric Models of Mortality

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Abstract

We consider a general problem of modeling a mortality law of a population of failing units with some parametric function. In this setting we define a mortality table of crude rates as a statistical estimator with multinomial distribution and show its consistency as well as asymptotic normality. We further derive the statistical properties of parameter estimators in a parametric mortality model based on a weighted square loss function. We use the obtained results to study consistency and appropriateness of the parametric bootstrap method in our setting. We derive the conditions on the assumed parametric mortality law and the loss function, under which the bootstrap is consistent for estimating the model parameters, their standard errors and corresponding confidence intervals. We apply our results to a model of Aggregate US Mortality Table based on a so called mixture of extreme value distributions suggested by Carriere (1992).

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Notes

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We realize that in many actuarial applications the data available for the construction of mortality rates is very different that used in Eq. (2.1). However, for the most part of the discussion presented in this paper the actual way of arriving at the values of the components of $q^{(n)}$ is irrelevant, as long as we can assume their asymptotic independence and normality (see Theorem 4.1). We have chosen to consider model given by Eq. (2.1) due to its simplicity as well as relevance to the general survival analysis theory.

Adjusting for this type of censoring would make a difference only for the relatively small sample sizes (n).

The resample size n is typically assumed to maximize the efficiency of the bootstrap. However, for very large n, like e.g., n=100 000 used in the US Mortality Table, it may be more practical to consider resamples of smaller size $m=m_n$. All our theoretical results on bootstrap consistency, carry over to this case, as long as $m_n \to\infty$ when $n\to\infty$. See, e.g., Bickel et al. (1997) for the discussion of these issues.

The results of this section show also the validity of the multivariate bootstrap approximation. For the analogues of the formulae (5.1) and (5.2) see, for instance, Shao and Tu (1995) and the references therein.

In order to maximize the efficiency of the procedure one should use here the same method of interpolation as the one used to create original values of the in the table, however, in our example we simply applied linear interpolation to obtain the values of the pseudo-empirical distribution at the integer ages and then to calculate the

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