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# Decay Properties for the Damped Wave Equation with Space Dependent Potential and Absorbed Semilinear Term

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## Abstract

We consider the Cauchy problem for the damped wave equation with space dependent potential $V(x)u_t$ and absorbed semilinear term $|u|^{\rho-1}u$ in $R^N$. Our assumption on $V(x) \sim (1+|x|^2)^{-\alpha/2}$ $(0 \le \alpha < 1)$ still implies the diffusion phenomena and the decay rates of solutions are expected to be the same as the corresponding parabolic problem. In this paper we obtain two kinds of decay rates of the solution effective for $\rho > \rho_c(N, \alpha) := 1 + 2/(N-\alpha)$ and for $\rho < \rho_c(N, \alpha)$. We believe that in the "supercritical" exponent the decay rates obtained are almost the same as those for the linear parabolic problem, while, in the "subcritical" exponent the solution decays faster than that of linear equation, thanks to the absorbed semilinear term. So we believe that $\rho_c(N, \alpha)$ is a

critical exponent. Note that $\rho_c(N, \alpha)$ with $\alpha = 0$ coincides to the Fujita exponent $\rho_F(N) := 1 + \frac{2}{N}$.

Keywords:

Absorbed semilinear term | Damped wave equation | Space dependent potential

Mathematics Subject Classification:

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