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Intra-day periodicity, temporal aggregation and time-to-maturity in FTSE-100 index futures volatility

David G. McMillan & Alan E. H. Speight

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Abstract

Intra-day periodicity has been widely observed in financial data. Recent research examining intra-day foreign exchange rate volatility dynamics reports that failure to account for this periodicity results in inconsistent GARCH parameter estimates in relationship to theoretical predictions on temporal aggregation. This article seeks to appraise the generality of this conclusion to the FTSE-100 index futures market. The nature of periodicity is first examined. Subsequent empirical results concerning the temporal aggregation of GARCH models show that the use of returns that are not adjusted for such periodicity are misleading. However, adjustment using a sine-cosine wave method or standardization by mean absolute returns provide more consistent results, the latter method dominating in out-of-sample forecasting of the volatility of successive individual futures contracts. The potential time-to-maturity effects of single

contracts are also considered, but are statistically rejected for both forms of periodicity-adjusted data.

Notes

[1] For reviews, see Bollerslev et al. (1992), Bera and Higgins (1993), and Bollerslev et. al. (1994).

[2] See, for example, Guillaume et al. (1995), Müller et al. (1996), Dacorogna et al. (1997) and Andersen and Bollerslev (1997a).

[3] See, for example, Wood et al. (1985), Harris (1986), McInish and Wood (1988) for evidence of a 'U'-shape pattern in the intra-day volatility of stock returns traded on the NYSE, Jain and Joh (1988) for the S&P 500 index, and Eckman (1992) for S&P 500 index futures.

[4] Andersen and Bollerslev (1997a), for example, found that such adjustment leads to improved GARCH model performance in their analysis of deutschmark–dollar exchange rate and S&P 500 stock index futures returns, but still note some discrepancy between estimated and theoretically implied GARCH coefficients under temporal aggregation. Also see Guillaume et al. (1995).

[5] For further elaboration of this issue see, for example, Andersen et al. (1999).

[6] Various explanations for why such intra-day patterns exist have been proposed in the context of models of market microstructure. The private information model of Admati and Pfleiderer (1988), following Kyle (1985), proposes that the 'U'-shape pattern is results from the concentration of privately informed trading when the activity of liquidity traders is greatest, typically at the beginning and end of the day. In contrast, Brock and Kleidon (1992) similarly predict that market turnover will be highest at the beginning and end of the trading day on the basis of a model of continuous trading subject to transaction costs and periodic market closure.

[7] This data period also extends and updates the September 1991–November 1991 sample analysed by Abhyankar et al. (1997), and the January 1991–June 1993 sample analysed by Abhyankar et al. (1999).

[8] Since multiple futures contract maturities are traded at any given time, some criterion must be set in order to obtain a continuous price series. The point at which the price data taken switches from one contract to another is frequently termed the rollover date. In the data analysed here, contract price splicing is conducted on the day prior to expiry, resulting in 14 rollover dates due to the quarterly expiry cycle of FTSE-100 futures in March, June, September and December. This is a standard method of constructing an continuous price series from high frequency futures data (Becker et al. 1995), and given that the contracts examined are heavily traded, such a sampling procedure should induce no systematic bias.

[9] Eighteen days were discovered to have been lost from the data-tape. These days were: 2 March 1992; 21 May 1992; 31 July 1992; 29 and 30 September 1992; 7 October 1992; 5–8, 23, 26 July 1993; and 7–10, 13, and 14 December 1993.

[10] An alternative approach employed in the literature is to take price data on the nearest maturity (front month) contract until the day that traded volume in that contract is exceeded by traded volume in the second nearest contract (e.g. Abhyankar et al., 1995). This procedure ensures that price data for the most liquid contract is always used. Inspection of traded volume for FTSE-100 futures contracts reveals that volume rollover was indeed on the day immediately prior to expiry in all but one case, and on the expiry date itself in the remaining case. The standard splicing criterion described in the text may therefore also be considered reasonable from a liquidity perspective.

[11] The standard errors of these statistics in their corresponding asymptotic normal distributions are given by $(6/T)^{1/2}$ and $(24/T)^{1/2}$, or 0.0088 and 0.0176 respectively (Jarque and Bera, 1987).

[12] This is in contrast to results reported for the absolute returns of intra-day deutschmark-US dollar exchange rates and S&P-500 stock index futures by Andersen and Bollerslev (1997a), for example. As those authors note, but do not investigate further, the increased size of autocorrelations at the five-day weekly frequency signals the presence of a day-of-the week-effect. The absence of any such effects here suggests the absence of any day-of-the-week anomaly in the FTSE-100 futures market. For a recent critique and reappraisal of the evidence for such anomalous calendar effects in stock returns, see Sullivan et al. (1998).

[13] Among alternative approaches, Baillie and Bollerslev (1991) suggested introducing a simple dummy variable for each intra-day interval, while Müller et al. (1990), Dacorogna et al. (1993) and Guillaume et al. (1995) suggested using time scale transformations, based on polynomial approximations to activity in distinct geographical regions over the twenty-four hour trading cycle. The former approach has the disadvantage of requiring a large number of additional parameters to be estimated at the higher intra-day frequencies. The innovative latter approach, while appealing in the context of the foreign exchange market, has little direct bearing in the context analysed here, and our attention therefore focuses on the more general adjustment procedures described in the text.

[14] The restriction $\rho<1$ is also required for stability and covariance stationarity of the error process, and as a necessary condition for the existence of a finite unconditional variance. As shown by Nelson (1990), the integrated-GARCH arising under $\rho = 1$ (Engle and Bollerslev, 1986) nevertheless remains strictly stationary and ergodic.

[15] In the terminology of Drost and Nijman, financial returns are modelled as 'flow' variables, such that $r_t = \Delta x_t$, where x_t, the (logarithmic) security price, is a 'stock' variable.

[16] While the first-order GARCH model is a widely preferred specification for the modelling of return volatility dynamics, and that specification corresponds to the class of models for which theoretical aggregation results are available, it does not necessarily provide the preferred specification at all intra-day frequencies. Moreover, that serial dependence in conditional mean will in general increase the order of the implied low frequency weak GARCH model beyond the order of the high frequency GARCH(1, 1) model. However, Nelson (1990, 1992) establishes general conditions under which first-order GARCH models will, even if misspecified, produce consistent estimates and satisfy the temporal aggregation convergence results given in the text, though it should be noted that those conditions are derived in the absence of any deterministic periodicity in the volatility process.

[17] Note that:

where β and α refer to the estimated high frequency parameters, and κ_ε to the high frequency level of kurtosis. Thus the parameters of the low frequency model depend not only upon the parameters of the high frequency model, but also on the high frequency kurtosis. The implied low frequency kurtosis level is given by:

while the relationship between the kurtosis of the rescaled innovations and is given by:

For further details, see Drost and Nijman, (1993), p. 920, Table 1and Equations 13–18 in particular.

[18] Among the various possible explanations for this paradoxical result, it has been suggested that intra-day volatility may be reflect the aggregation of numerous volatility components, each of which is endowed with a particular dependence structure due to the arrival of heterogeneous information flows (Andersen and Bollerslev, 1997b). If the decay of the short-run component(s) dominates over inter-day frequencies, and the long-run volatility component(s) dominate over inter-day and lower frequencies, the aggregation of such component processes will give rise to the near-integrated dependencies commonly found in the inter-day volatilities of many asset prices. For further discussion of this and related approaches, see Andersen and Bollerslev (1997b) and Müller et al. (1997).