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# Valuing the Guaranteed Minimum Death Benefit Clause with Partial Withdrawals

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## Abstract

In this paper, we give a method for computing the fair insurance fee associated with the guaranteed minimum death benefit (GMDB) clause included in many variable annuity contracts. We allow for partial withdrawals, a common feature in most GMDB contracts, and determine how this affects the GMDB fair insurance charge. Our method models the GMDB pricing problem as an impulse control problem. The resulting quasi-variational inequality is solved numerically using a fully implicit penalty method. The numerical results are obtained under both constant volatility and regime-switching models. A complete analysis of the numerical procedure is included. We show that the discrete equations are stable, monotone and consistent and hence obtain convergence to the unique, continuous viscosity solution, assuming this exists. Our results show that the addition of the partial withdrawal feature significantly increases the fair insurance charge for GMDB contracts.

Key Words:

- Variable annuities
- guaranteed minimum death benefit (GMDB)
- viscosity solution
- impulse control
- fully implicit penalty method

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Notes

- <sup>1</sup> Intuitively, this can be viewed as a discretely observed lookback option based on the maximum value of the underlying (Wilmott, [1998](#)).
- <sup>2</sup> We remark that our PDE approach can easily be extended to model different withdrawal policies. For example, an alternate withdrawal policy, whereby the deposit is reduced by the amount withdrawn but the death benefit is reduced on a proportional basis, could be easily implemented.
- <sup>3</sup> Note that this is trivially true at .

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