Applied Mathematical Finance ›
Volume 21, 2014 - Issue 2

Original Articles

Prices and Asymptotics for Discrete Variance Swaps

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Pages 140-173 | Received 22 Jul 2012, Published online: 01 Aug 2013

Cite this article https://doi.org/10.1080/1350486X.2013.820524 Check for updates

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Abstract

We study the fair strike of a discrete variance swap for a general time-homogeneous stochastic volatility model. In the special cases of Heston, Hull–White and Schöbel–Zhu stochastic volatility models, we give simple explicit expressions (improving Broadie and Jain (2008a). The effect of jumps and discrete sampling on volatility and variance swaps. International Journal of Theoretical and Applied Finance, 11(8), 761–797) in the case of the Heston model). We give conditions on parameters under which the fair strike of a discrete variance swap is higher or lower than that of the continuous variance swap. The interest rate and the correlation between the underlying price and its volatility are key elements in this analysis. We derive asymptotics for the discrete variance swaps and compare our results with those of Broadie and Jain (2008a. The effect of jumps and discrete sampling on volatility and variance swaps. International Journal of Theoretical and Applied Finance, 11(8), 761–797), Jarrow et al. (2013.

Discretely sampled variance and volatility swaps versus their continuous approximations. Finance and Stochastics, 17(2), 305–324) and Keller-Ressel and Griessler (2012. Convex order of discrete, continuous and predictable quadratic variation and applications to options on variance. Working paper. Retrieved from http://arxiv.org/abs/1103.2310).

Key Words:

Discrete variance swap | Heston model | Hull–White model | Schöbel–Zhu model

View correction statement:

Erratum

Acknowledgements

Carole Bernard acknowledges support from the Natural Sciences and Engineering Research Council of Canada. Zhenyu Cui acknowledges support from the Bank of Montreal Capital Markets Advanced Research Scholarship. The authors thank Jinyuan Zhang for her help as research assistant, as well as seminar participants Christa Cuchiero, Olympia Hadjiliadis, Antoine Jacquier, Adam Kolkiewicz, Roger Lee, Don McLeish, Johannes Ruf, David Saunders and Michael Suchanecki for helpful suggestions. We are particularly grateful to an anonymous referee for his/her constructive and helpful comments on an earlier draft of this paper.

Notes

1. [1] See, for example, Howison, Rafailidis, and Rasmussenn (2004), Windcliff, Forsyth, and Vetzal (2006), Benth, Groth, and Kufakunesu (2007) and Broadie and Jain (2008b).

2. [2] These conditions ensure that we can apply Fubini's theorem to exchange the order of integration. They are easily verified in specific examples.

3. [3] Thanks to the anonymous referee for pointing out this general expression.

4. [4] The impact of stochastic interest rates on variance swaps is studied by Hörfelt and Torné (2010). Long-dated variance swaps will usually be sensitive to the interest rate volatility.

5. [5] This formula has been implemented in Matlab and its code is available at http://www.runmycode.org/CompanionSite/Site135 or upon request from authors as well as for all other formulas that appear in this paper.

6. [6] We shall note that here $m(V_t) = V_t$ (where $m(\cdot)$ is defined in (1)) instead of , thus the process V_t models the volatility and not the variance. In particular, in the Schöbel–Zhu model, the variance process follows .

7. [7] See Proposition 6.7 for an explicit expansion.

8. [8] See Definition 2.6 on p. 112 of Keller-Ressel and Muhle-Karbe (in press).

9. [9] This can be easily seen from the fact that for all , , and note that here .

10. [10] It reduces to studying the sign of . It is an increasing function of , so it is larger than , which is always positive because its minimum is 0 obtained when .

11. [11] For the two sets of parameters above, we compute the critical interest rate as defined in Remark 2.1. Set 1: ; Set 2: , and we can see that the interest rates are both larger than .

12. [12] The notation V_t in the Schöbel–Zhu model corresponds to the square root of what is denoted by V_t in the Heston model.

13. [13] Note that in terms of our notation, the parameters in Hurd and Kuznetsov (2008) and our parameters have the correspondence , , .

14. [14] Note that is a decreasing function in t, thus is sufficient for for all $i = 0, 1, \ldots, n$.