The European Journal of Finance ›

Volume 12, 2006 - Issue 6-7: Forecasting Financial Markets

Original Articles

Small sample properties of GARCH estimates and persistence

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Pages 473-494 | Published online: 28 Nov 2010

Cite this article https://doi.org/10.1080/13518470500039436

Abstract

It is shown that the ML estimates of the popular GARCH(1,1) model are significantly negatively biased in small samples and that in many cases converged estimates are not possible with Bollerslev's non-negativity conditions. Results also indicate that a high level of persistence in GARCH(1,1) models obtained using a large number of observations has autocorrelations lower than these ML estimates suggest in small samples. Considering the size of biases and convergence errors, it is proposed that at least 250 observations are needed for ARCH(1) models and 500 observations for GARCH(1,1) models. A simple measure of how much GARCH conditional volatility explains squared returns is proposed. The measure indicates that for a typical index return volatility whose ARCH parameter is very small, the conditional volatility hardly explains squared returns.

Keywords:

Small sample volatility GARCH persistence

Acknowledgements

We would like to thank Gordon Gemmill, Mark Salmon, Stephen E. Satchell, and the participants of the 2003 Forecasting Financial Markets Conference for their helpful comments. The second author acknowledges the financial support from CNPq, Ibmec Business School and PRONEX. All remaining errors are our responsibilities.

Notes

[1]See Drost and Nijman (1993) for further discussion of this topic.

[2]The general properties of small sample estimates in GARCH models are known in econometrics—Engle et al. (1985), Bollerslev (1988), Diebold and Pauly (1989), and Baillie and Chung (2001). However, they are not well known in applied areas such as finance and economics. In addition, the effects of Bollerslev's (1986) non-negativity conditions on bias and convergence errors in small samples are rarely investigated. Many variations of GARCH models (IGARCH, EGARCH, GARCH with t-distribution, etc.) could be used. However, we believe that the estimates of these variations would show similar properties in small samples to those of the standard ARCH and GARCH models.

[3]We also tried {0.7, 0.05}, {0.3, 0.05}, {0.05, 0.7}, and {0.05, 0.3} with Bollerslev's non-negativity conditions, but did not find any local maxima.

[4]We only consider the cases that the standardized residuals are normal. Simulations with other probability density functions could be considered. See Baillie and Chung (2001), for example.

[5]We used a few different algorithms to see if there were any changes in convergence errors, but found little difference.

[6]We also estimated ARCH(1) and GARCH(1,1) models for the FTSE100 daily log-returns using the same method as described in this section and found similar results. The results are not reported here, but can be obtained from the authors upon request.

[7]See Nelson (1991), who raises this point.

[8]This is a reason why unit root tests are affected by the presence of MA components; see the simulation results of Phillips and Perron (1988) and Schwert (1989).

[9]The same experiments as those for GARCH models were carried out for SV models for the S&P500 index log-squared returns. The results are consistent with what we found in table 2 and can be obtained from authors by request.

[10]Andersen and Bollerslev (1998) calculate the population R^2 of the ex-post squared return – GARCH(1,1) volatility regression which is $\alpha^2/(1 - \beta^2 - 2\alpha\beta)$. Their results are not different from Theorem 1 in the sense that for realistic parameter values of α and β, R^2 is very low and GARCH(1,1) does not explain squared returns well.

[11]See Equation 4 in the previous section for the autocorrelations of ARCH(1) and GARCH(1,1) models.