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Delta-hedging vega risk?

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Abstract

In this article we compare the profit and loss arising from the delta-neutral dynamic hedging of options, using two possible values for the delta of the option. The first is the Black–Scholes implied delta, while the second is the local delta, namely the delta of the option in a generalized Black–Scholes model with a local volatility, recalibrated to the market smile every day. We explain why, in negatively skewed markets, the local delta should provide a better hedge than the implied delta during slow rallies or fast sell-offs, and a worse hedge, although to a smaller extent, during fast rallies or slow sell-offs. Since slow rallies and fast sell-offs are more likely to occur than fast rallies or slow sell-offs in negatively skewed markets (provided we have physical as well as implied negative skewness), we conclude that, on average, the local delta provides a better hedge than the implied delta in negatively skewed markets. We obtain the same conclusion in the case of positively skewed markets. We illustrate these results using both simulated and real time-series of equity-index data, which have had a large negative implied skew since the stock market crash of October 1987. Moreover, we

check numerically that the conclusions we draw are true when transaction costs are taken into account. In the last section we discuss the case of barrier options.

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Notes

Vähämaa [55] assesses the significance of the differences by a bootstrapping method with 1000 resamplings.

Note the difference between these conclusions and the implications of section 2.2 (see also the discussion in section 3.5).

We shall take τ as being equal to one market day in the numerical experiments of sections 4 and 5.

International Financial Futures and Options Exchange, http://www.liffe-data.com.

Deutsche Terminbörse, http://deutsche-boerse.com/INTERNET/EXCHANGE/index_e.htm.

As explained in appendix B, these prices are computed by an implicit finite difference scheme which is exactly solved by the Gauss algorithm, hence the label 'Gauss' for these curves.

The last column in table 4 illustrates results concerning a barrier option that will be considered in section 5.