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The risk-shifting effect and the value of a warrant

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Abstract

The exercise of a warrant leads to the well-known dilution phenomenon, the effects of which have been extensively studied over the last four decades. In contrast, the existing literature has paid inadequate attention to the volatility spillover between stockholders and warrant holders. This ‘risk-shifting effect’ has significant implications for warrant pricing, since any formula that assumes a constant volatility of stock returns produces a bias. In this paper we show that a CEV process with a specific elasticity parameter properly models the stochastic volatility of stock returns for a firm with warrants outstanding. In addition, we propose an approximate analytical formula, exclusively based on observable market variables, that is able to absorb the risk-shifting bias.

Keywords:

Corporate finance Applied finance Corporate risk management Corporate valuation

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Notes

§Similarly, the non-stationary nature of stock volatility due to the presence of debt or warrants was also studied by Geske (1979) and Bensoussan et al. (1994, 1995).

†By 'leverage effect' the financial literature refers to a negative correlation between stock prices and volatilities caused by the presence of debt financing (Black 1976).

‡For instance, setting arbitrarily the elasticity parameter to 0.5 would lead to a closed formula for the call price (Beckers 1980).

†The absence of debt makes the stochastic process in equation (1) govern both the asset and equity value.

‡N(·) denotes the normal cumulative distribution function, and

†To denote that $\sigma_s(t, a_t)$ is known at time t, we omit the argument and we write σ_s.

‡This is a first-order approximation, since we ignore powers of price increment greater than one.

§Shown in appendix A.

¶N′(·) denotes the normal density function, and

⊥Intuitively, the net effect on ϵ_{σ_s, s^*} of this substitution is negligible once we recall equation (5). Both of these substitutions (s_t for a_t and σ_s for σ) produce a downward approximation of the variables, which lowers the product σa_t but also increases the expression in brackets. As a matter of fact, the above-mentioned approximation determines a drop in $N(d_1)$ (the warrant's delta) due to the decrease in both moneyness and volatility. Hence, the suggested replacement causes two opposite outcomes, whose net effect tends to be insignificant. The analytical expression of the difference between the two elasticities is provided in appendix A.

†This value is also computable through the numerical algorithm proposed by Ukhov (2004). Nevertheless, in our simulation this method is not necessary since we assume that the asset value and its volatility are both known.

‡We obtain a_t applying the Newton–Raphson algorithm to the process defined in equation (4).

§We consider a log-normal distribution of asset values. The moneyness bounds are computed according to the following probability intervals: DOTM [0.01, 0.20), OTM [0.20, 0.45), ATM [0.45, 0.55), ITM [0.55, 0.80), and DITM [0.80, 0.99]. First and last percentiles are excluded to avoid an infinite support.

†Simulations based on different values of σ do not produce significant changes in terms of pricing accuracy. Tables will be provided by the authors upon request.