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Geometric or Arithmetic Mean: A Reconsideration

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Abstract

An unbiased forecast of the terminal value of a portfolio requires compounding of its initial value at its arithmetic mean return for the length of the investment period. Compounding at the arithmetic average historical return, however, results in an upwardly biased forecast. This bias does not necessarily disappear even if the sample average return is itself an unbiased estimator of the true mean, the average is computed from a long data series, and returns are generated according to a stable distribution. In contrast, forecasts obtained by compounding at the geometric average will generally be biased downward. The biases are empirically significant. For investment horizons of 40 years, the difference in forecasts of cumulative performance can easily exceed a factor of 2. And the percentage difference in forecasts grows with the investment horizon, as well as with the imprecision in the estimate of the mean return. For typical investment horizons, the proper compounding rate is in between the arithmetic and geometric values.

An unbiased forecast of the terminal value of a portfolio requires the initial value to be compounded at the arithmetic mean rate of return for the length of the investment period. An upward bias in forecasted values results, however, if one estimates the mean return with the sample average and uses that average to compound forward. This bias arises because cumulative performance is a nonlinear function of average return and the sample average is necessarily a noisy estimate of the population mean. Surprisingly, the bias does not necessarily disappear asymptotically, even if the sample average is computed from long data series and returns come from a stable distribution with no serial correlation. Instead, the bias depends on the ratio of the length of the historical estimation period to that of the forecast period.

Forecasts obtained by compounding at the geometric average will generally be downwardly biased.

Therefore, for typical investment horizons, the proper compounding rate is in between the arithmetic and geometric rates. Specifically, unbiased estimates of future portfolio value require that the current value be compounded forward at a weighted average of the two rates. The proper weight on the geometric average equals the ratio of the investment horizon to the sample estimation period. Thus, for short investment horizons, the arithmetic average will be close to the “unbiased compounding rate.” As the horizon approaches the length of the estimation period, however, the weight on the geometric average approaches 1. For even longer horizons, both the geometric and arithmetic average forecasts will be upwardly biased.

The implications of these findings are sobering. A consensus is already emerging that the 1926–2002 historical average return on broad market indexes, such as the S&P 500 Index, is probably higher than likely future performance. Our results imply that the best forecasts of compound growth rates for future investments are even lower than the estimates emerging from the research behind this consensus.

The choice of compounding rate can have a dramatic impact on forecasts of future portfolio value. Compounding at the arithmetic average return calculated from sample periods of either the most recent 77 or 52 years results in forecasts of future value for a sample of countries that are roughly double the corresponding unbiased forecasts based on the same data periods. Indeed, for reasonable risk and return parameters, at investment horizons of 40 years, the differences in forecasts of total return generally exceed a factor of 2.

The percentage differences between unbiased forecasts versus forecasts obtained by compounding arithmetic or geometric average returns increase with the ratio of the investment horizon to the sample estimation period as well as with the imprecision in the estimate of the mean return. For this reason, emerging markets present the greatest forecasting problem. These markets have particularly short historical estimation periods and return histories that are particularly noisy. For these markets, therefore, the biases we analyzed can be especially acute. Even for developed economies, however, with their longer histories, bias can be significant if one disregards data from very early periods.

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