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## Asian options on the harmonic average

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### 1. Introduction

The contracts written on the harmonic average of the underlying price are quite popular in the foreign exchange market. If  $X$  denotes the foreign currency and  $Y$  denotes the domestic currency, the pay-off of the contract is a function of a price of an asset  $H$  which is defined as

$$H(T) = \left[ \int_0^T [X_Y(t)]^{-1} \eta(t) dt \right]^{-1} Y(T) \left[ \frac{1}{\int_0^T X_Y(t) \eta(t) dt} \right] Y(T).$$

The harmonic average resembles a quanto option: the price  $YX(t)$  is monitored with respect to the foreign currency  $X$ , but the pay-off is settled in the domestic currency  $Y$ . Although the pricing problem appears to be rather complex, it can be ultimately simplified to a partial differential equation in one spatial variable after a numeraire change and using the time reversal argument.

Let us first introduce notation that we use more generally in this article. By  $X$  or  $Y$  we mean an asset rather than the price of the asset. One can think about  $X$  or  $Y$  as names of the assets that have no numerical meaning. We write  $X(t)$  or  $Y(t)$

in the situation when the asset is required at time  $t$  for trading, hedging or settling a financial contract. The price of an asset is a pairwise relationship of two assets, which we denote by  $X_Y(t)$ : the number of assets  $Y$  required to obtain a unit of an asset  $X$ . The asset  $Y$  is known as a reference asset or as a numeraire. We will also use the relationship

$$X_Z(t) = X_Y(t) \cdot Y_Z(t),$$

known as the change of numeraire formula. We will write  $X(t) = Y(t)$  in terms of the assets if  $X$  and  $Y$  have the same price (numeraire independent result). Similarly  $X(t) > Y(t)$  means that the asset  $X$  has a larger price than the asset  $Y$ .

Given two assets  $X$  and  $Y$ , several types of averages can be considered:

$$\text{Arithmetic: } A(T) = \left[ \int_0^T X_Y(t) \eta(t) dt \right] Y(T), \quad (1.1)$$

$$\text{Geometric: } G(T) = \left[ \exp \left( \int_0^T \log[X_Y(t)] \eta(t) dt \right) \right] Y(T), \quad (1.2)$$

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