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Asian options on the harmonic average

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1. Introduction

The contracts written on the harmonic average of the underlying price are quite popular in the foreign exchange market. If X denotes the foreign currency and Y denotes the domestic currency, the pay-off of the contract is a function of a price of an asset H which is defined as

$$H(T) = \left[\int_0^T [X_Y(t)]^{-1}\eta(t)dt\right]^{-1} Y(T) \quad \left[\frac{1}{\int_0^T X_Y(t)\eta(t)dt}\right] Y(T).$$

The harmonic average resembles a quanto option: the price YX (t) is monitored with respect to the foreign currency X, but the pay-off is settled in the domestic currency Y. Although the pricing problem appears to be rather complex, it can be ultimately simplified to a partial differential equation in one spatial variable after a numeraire change and using the time reversal argument.

Let us first introduce notation that we use more generally in this article. By X or Y we mean an asset rather than the price of the asset. One can think about X or Y as names of the assets that have no numerical meaning. We write $X(t)$ or $Y(t)$ in the situation when the asset is required at time t for trading, hedging or settling a financial contract. The price of an asset is a pairwise relationship of two assets, which we denote by $X_Y(t)$: the number of assets Y required to obtain a unit of an asset X. The asset Y is known as a reference asset or as a numeraire. We will also use the relationship

$$X_Z(t) = X_Y(t) \cdot Y_Z(t),$$

known as the change of numeraire formula. We will write $X(t) = Y(t)$ in terms of the assets if X and Y have the same price (numeraire independent result). Similarly $X(t) > Y(t)$ means that the asset X has a larger price than the asset Y.

Given two assets X and Y, several types of averages can be considered:

$$\text{Arithmetic:} \quad A(T) = \left[\int_0^T X_Y(t)\eta(t)dt\right] Y(T), \tag{1.1}$$

$$\text{Geometric:} \quad G(T) = \left[\exp\left(\int_0^T \log[X_Y(t)]\eta(t)dt\right)\right] Y(T), \tag{1.2}$$

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