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Using Duration and Convexity in the Analysis of Callable Convertible Bonds

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Dunetz and Mahoney derived measures of duration and convexity for callable bonds. This note extends their analysis to derive duration and convexity measures for callable convertible bonds.

Duration measures the economic life of a bond by weighting all its cash flows relative to the time in which each is paid.[1] Given a bond's maturity date, coupon rate, par value and yield to maturity, one can calculate the price of a bond and its duration. Macaulay's duration (MD) is calculated as follows:

Eq. 1

$$MD = \left\{ \sum_{t=1}^{N} \frac{t(C_t)}{(1 + YTM)^t} + \frac{N(PAR)}{(1 + YTM)^N} \right\} / P$$

where

t = time period,
YTM = yield to maturity,
C_t = cash flow at time t,
N = maturity date and
P = price of bond.

Another method of calculating duration uses a Taylor-series expansion to determine the rate of change in bond price with a given change in yield. A Taylor series applied to the price-yield relationship can be expressed as follows:

Eq. 2

$$\frac{\Delta P}{P} = \frac{1}{P}\frac{\partial P}{\partial Y}(\Delta Y) + \frac{1}{2!P}\frac{\partial^2 P}{\partial Y^2}(\Delta Y)^2 + \ldots$$

Using the Taylor-series expansion, modified duration is the first derivative divided by −P:

Eq. 3

$$D = \frac{-1}{P}\frac{\partial P}{\partial Y}$$

and **convexity** is the second-order derivative divided by 2P:

Eq. 4

$$C = \frac{1}{2P}\frac{\partial^2 P}{\partial Y^2}.$$

Modified duration (Equation (3)) implies that the bond price and yield relationship can be estimated by an inverse linear relationship. However, the true relationship between price and yield is not linear, but curvilinear; hence the need for convexity. Convexity measures the curvature of the price-yield relationship (see Figure A).

An investor should not necessarily be indifferent between two bonds with similar maturities and present values (price). The bonds' durations and convexities may be critical. Given two bonds with the same price and maturity, an investor would normally prefer the bond with the lower duration. If their durations were also similar, an investor would prefer the bond with the higher convexity.

Applications of duration and convexity have been extended to financial instruments other than the traditional straight bond. Measures of interest-rate sensitivity have become instrumental in evaluating portfolios, projects and overall assets and liabilities. Duration and convexity have also been used to evaluate individual securities such as stocks, which theoretically have an infinite maturity. Variations of the straight bond—bonds with indefinite but bounded maturities—have also been evaluated with duration and convexity. This note focuses on one such bond variation—the callable, convertible bond. Before examining this variant, we reexamine the case of the callable bond.

Duration and Convexity of Callable Bonds

Duration and convexity have been applied to callable bonds.[2] One can measure the duration and convexity of callable bonds in three ways. (1) Ignore the call feature and value the bonds using maturity dates. (2) If the bond sells at a discount, value it using the maturity date, but if it sells at a premium, value it using the call date. (3) Break the callable bond into two components as follows:

Eq. 5

$$P_{CB} = P_{SB} - CO$$

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