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Abstract:

Arithmetic Asian options are difficult to price and hedge as they do not have closed-form analytic solutions. The main theoretical reason for this difficulty is that the payoff depends on the *finite sum* of correlated lognormal variables, which is not lognormal and for which there is no recognizable probability density function. We use elementary techniques to derive the probability density function of the *infinite sum* of correlated lognormal random variables and show that it is reciprocal gamma distributed, under suitable parameter restrictions. A random variable is reciprocal gamma distributed if its inverse is gamma distributed. We use this result to approximate the *finite sum* of correlated lognormal variables and then value arithmetic Asian options using the reciprocal gamma distribution as the state-price density function. We thus obtain a closed-form analytic expression for the value of an arithmetic Asian option, where the cumulative density function of the gamma distribution, G(d) in our formula, plays the exact same role as N(d) in the Black-Scholes/Merton formula. In addition to being theoretically justified and exact in the limit, we compare our method against other algorithms in the literature and show our method is quicker, at least as accurate, and, in our opinion, more intuitive and pedagogically appealing than any previously published result, especially when applied to high yielding currency options.

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