Abstract

In this paper we present an arbitrage pricing framework for valuing and hedging contingent equity index claims in the presence of a stochastic term and strike structure of volatility. Our approach to stochastic volatility is similar to the Heath-Jarrow-Morton (HJM) approach to stochastic interest rates. Starting from an initial set of index options prices and their associated local volatility surface, we show how to construct a family of continuous time stochastic processes which define the *arbitrage-free* evolution of this local volatility surface through time. The no-arbitrage conditions are similar to, but more involved than, the HJM conditions for arbitrage-free stochastic movements of the interest rate curve. They guarantee that even under a general stochastic volatility evolution the initial options prices, or their equivalent Black–Scholes implied volatilities, remain fair.

We introduce stochastic implied trees as discrete implementations of our family of continuous time models. The nodes of a stochastic implied tree remain fixed as time passes. During each discrete time step the index moves randomly from its initial node to some node at the next time level, while the local transition probabilities between the nodes also vary. The change in transition probabilities corresponds to a general (multifactor) stochastic variation of the local volatility surface. Starting from any node, the future movements of the index and the local volatilities must be restricted so that the transition probabilities to all future nodes are simultaneously martingales. This guarantees that initial options prices remain fair. On the tree, these martingale conditions are effected through appropriate choices of the drift parameters for the transition probabilities at every future node, in such a way that the subsequent evolution of the index and of the local volatility

surface do not lead to riskless arbitrage opportunities among different option and forward contracts or their underlying index.

You can use stochastic implied trees to value complex index options, or other derivative securities with payoffs that depend on index volatility, even when the volatility surface is both skewed and stochastic. The resulting security prices are consistent with the current market prices of all standard index options and forwards, and with the absence of future arbitrage opportunities in the framework. The calculated options values are independent of investor preferences and the market price of index or volatility risk. Stochastic implied trees can also be used to calculate hedge ratios for any contingent index security in terms of its underlying index and all standard options defined on that index.
